1.) (a) correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ A2 N2 2

$$\boldsymbol{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$$

(b) (i) attempt to substitute $t = 2$ into the equation (M1)

e.g. $\begin{pmatrix} 2 \\ 6 \\ -4 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$

$\overrightarrow{\mathrm{OP}} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$ 3 A1 N2

(ii) correct substitution into formula for magnitude A1

e.g. $\sqrt{3^2 + 5^2 + (-2)^2}, \sqrt{3^2 + 5^2 + 2^2}$

$\left|\overrightarrow{\mathrm{OP}}\right| = \sqrt{38}$ A1 N14

[6]

2.) (a) (i) $\overrightarrow{\mathrm{BA}} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$ A1 N1

(ii) evidence of combining vectors (M1)

e.g. $\overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BC}} = \overrightarrow{\mathrm{AC}}, \overrightarrow{\mathrm{BA}} + \overrightarrow{\mathrm{AC}}, \begin{pmatrix} -2 \\ 2 \\ -6 \end{pmatrix} - \begin{pmatrix} -3 \\ 0 \\ -4 \end{pmatrix}$

$\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ A1 N23

(b) (i) **METHOD 1**

finding $\overrightarrow{\mathrm{BA}} \cdot \overrightarrow{\mathrm{BC}}, \left|\overrightarrow{\mathrm{BA}}\right|, \left|\overrightarrow{\mathrm{BC}}\right|$ (A1)(A1)(A1)

e.g. $\overrightarrow{\mathrm{BA}} \cdot \overrightarrow{\mathrm{BC}} = 3\times1 + 0 + 4\times -2, \left|\overrightarrow{\mathrm{BA}}\right| = \sqrt{3^2 + 4^2}, \left|\overrightarrow{\mathrm{BC}}\right| = 3$

substituting into formula for cos ¯ M1

e.g. $\dfrac{3\times1 + 0 + 4\times -2}{3\sqrt{3^2 + 0 + 4^2}}, \dfrac{-5}{5\times3}$

$\cos\ \mathrm{ABC} = \dfrac{-5}{15}\left(= -\dfrac{1}{3}\right)$ A1 N3

METHOD 2

finding $\overrightarrow{AC}, \left|\overrightarrow{BA}\right|, \left|\overrightarrow{BC}\right|$ (A1)(A1)(A1)

e.g. $\left|\overrightarrow{AC}\right| = \sqrt{2^2 + 2^2 + 6^2}, \left|\overrightarrow{AB}\right| = \sqrt{3^2 + 4^2}, \left|\overrightarrow{BC}\right| = 3$

substituting into cosine rule M1

e.g. $\frac{5^2 + 3^2 - \left(\sqrt{44}\right)^2}{2 \times 5 \times 3}, \frac{25 + 9 - 44}{30}$

cos $ABC = -\frac{10}{30}\left(= -\frac{1}{3}\right)$ A1 N3

(ii) evidence of using Pythagoras (M1)

e.g. right-angled triangle with values, $\sin^2 x + \cos^2 x = 1$

sin $ABC = \frac{\sqrt{8}}{3}\left(= \frac{2\sqrt{2}}{3}\right)$ A1 N27

(c) (i) attempt to find an expression for $\overrightarrow{CD}$ (M1)

e.g. $\sqrt{(-4)^2 + 5^2 + p^2}, \left|\overrightarrow{CD}\right|^2 = 4^2 + 5^2 + p^2$

correct equation A1

e.g. $\sqrt{(-4)^2 + 5^2 + p^2} = \sqrt{50}, 4^2 + 5^2 + p^2 = 50$

$p^2 = 9$ A1

$p = 3$ AG N0

(ii) evidence of scalar product (M1)

e.g. $\begin{pmatrix} -4 \\ 5 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \overrightarrow{CD} \bullet \overrightarrow{BC}$

correct substitution

e.g. $-4 \times 1 + 5 \times 2 + 3 \times -2, -4 + 10 - 6$ A1

$\overrightarrow{CD} \bullet \overrightarrow{BC} = 0$ A1

$\overrightarrow{CD}$ is perpendicular to $\overrightarrow{BC}$ AG N06

[16]

3.) (a) evidence of appropriate approach (M1)

e.g. $\overrightarrow{AC} - \overrightarrow{AB}, \begin{pmatrix} 4 & -3 \\ 4 & -1 \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ A1 N22

(b) **METHOD 1**

$\overrightarrow{AD} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ (A1)

correct approach A1

e.g. $\overrightarrow{AD} - \overrightarrow{AB}, \begin{pmatrix} 1-3 \\ 3-1 \end{pmatrix}$

$\overrightarrow{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ AG N02

METHOD 2

Recognizing $\overrightarrow{CD} = \overrightarrow{BA}$ (A1)

correct approach A1

e.g. $\overrightarrow{BC} + \overrightarrow{CD}, \begin{pmatrix} 1-3 \\ 3-1 \end{pmatrix}$

$\overrightarrow{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ AG N02

(c) **METHOD 1**

evidence of scalar product (M1)

e.g. $\overrightarrow{BD} \bullet \overrightarrow{AC}, \begin{pmatrix} -2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

correct substitution A1

e.g. (–2)(4) + (2)(4), –8 + 8

$\overrightarrow{BD} \bullet \overrightarrow{AC} = 0$ A1

therefore vectors $\overrightarrow{BD}$ and $\overrightarrow{AC}$ are perpendicular AG N03

METHOD 2

attempt to find angle between two vectors (M1)

e.g. $\frac{\mathbf{a} \bullet \mathbf{b}}{\mathbf{ab}}$

correct substitution A1

e.g. $\frac{(-2)(4)+(2)(4)}{\sqrt{8}\sqrt{32}}, \cos - = 0$

$- = 90^\circ$ A1

therefore vectors $\overrightarrow{BD}$ and $\overrightarrow{AC}$ are perpendicular AG N0

[7]

4.) (a) appropriate approach (M1)

e.g. $\overrightarrow{AD} + \overrightarrow{OB}, B - A$

$\overrightarrow{AB} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ A1 N22

(b) **any** correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ A2 N22

where $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

e.g. $\boldsymbol{r} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \boldsymbol{r} = \begin{pmatrix} 2+t \\ -2-t \\ 5+t \end{pmatrix}, \boldsymbol{r} = 2\boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k} + t(\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k})$

(c) choosing correct direction vectors $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ (A1)(A1)

finding scalar product and magnitudes (A1)(A1)(A1)

scalar product $= 1 \times 2 + -1 \times 1 + 1 \times 3$ $(= 4)$

magnitudes $\sqrt{1^2 + (-1)^2 + 1^2}\ (= 1.73\ldots),\ \sqrt{4+1+9}\ (= 3.74\ldots)$

substitution into $\dfrac{\boldsymbol{u}.\boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}$ $\left(\text{accept}^{-} = \dfrac{\boldsymbol{u}.\boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}, \text{but not } \sin^{-} = \dfrac{\boldsymbol{u}.\boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}\right)$ M1

e.g. $\cos^{-} = \dfrac{1 \times 2 + -1 \times 1 + 1 \times 3}{\sqrt{1^2 + (-1)^2 + 1^2}\sqrt{2^2 + 1^2 + 3^2}}, \cos^{-} = \dfrac{4}{\sqrt{42}}$

$\theta = 0.906$ (51.9°) A1 N57

(d) **METHOD 1** $\left(\text{from } \boldsymbol{r} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}\right)$

appropriate approach (M1)

e.g. $\boldsymbol{p} = \boldsymbol{r}$, $\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}t = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}s$,

two **correct** equations A1A1

e.g. $1 + t = 2 + 2s$, $-1 - t = 4 + s$, $4 + t = 7 + 3s$

attempt to solve (M1)

one correct parameter A1

e.g. $t = -3$, $s = -2$

C is $(-2, 2, 1)$ A1 N36

METHOD 2 $\left(\text{from } \boldsymbol{r} = \begin{pmatrix} 2 \\ -2 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}\right) 1$

appropriate approach (M1)

$e.g.\ \boldsymbol{p} = \boldsymbol{r}\ \begin{pmatrix} 2 \\ -2 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} s,$

two **correct** equations A1A1

e.g. $2 + t = 2 + 2s$, $-2 - t = 4 + s$, $5 + t = 7 + 3s$

attempt to solve (M1)

one correct parameter A1

e.g. $t = -4$, $s = -2$

C is $(-2, 2, 1)$ A1 N36

[17]

5.) (a) any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter) A2 N2

$e.g.\ \boldsymbol{r} = \begin{pmatrix} -8 \\ -5 \\ 25 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$

Note: *Award A1 for* $\boldsymbol{a} + t\boldsymbol{b}$, *A1 for* $L = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** *for* $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

(b) recognizing scalar product must be zero (seen anywhere) R1
e.g. $\boldsymbol{a} \bullet \boldsymbol{b} = 0$

evidence of choosing direction vectors $\begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}, \begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix}$ (A1)(A1)

correct calculation of scalar product (A1)
e.g. $2(-7) + 1(-2) - 8k$

simplification that clearly leads to solution A1
e.g. $-16 - 8k$, $-16 - 8k = 0$

$k = -2$ AGN0

(c) evidence of equating vectors (M1)

$e.g.\ L_1 = L_3,\ \begin{pmatrix} -3 \\ -3 \\ -25 \end{pmatrix} + p\begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + q\begin{pmatrix} -7 \\ -2 \\ -2 \end{pmatrix}$

any **two** correct equations A1A1
e.g. $-3 + 2p = 5 - 7q$, $-1 + p = -2q$, $-25 - 8p = 3 - 2q$

attempting to solve equations (M1)
finding **one** correct parameter ($p = -3$, $q = 2$) A1

the coordinates of A are $(-9, -4, -1)$ A1N3

(d) (i) evidence of appropriate approach (M1)

$e.g.\ \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB},\ \overrightarrow{AB} = \begin{pmatrix} -8 \\ -5 \\ 25 \end{pmatrix} - \begin{pmatrix} -9 \\ -4 \\ -1 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 1 \\ -1 \\ 26 \end{pmatrix}$ A1 N2

(ii) finding $\overrightarrow{AC} = \begin{pmatrix} 7 \\ 2 \\ 2 \end{pmatrix}$ A1

evidence of finding magnitude (M1)

e.g. $\left|\overrightarrow{AC}\right| = \sqrt{7^2 + 2^2 + 2^2}$

$\left|\overrightarrow{AC}\right| = \sqrt{57}$ A1N3

[18]

6.) (a) evidence of appropriate approach (M1)

e.g. $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}, \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} -8 \\ -1 \\ -1 \end{pmatrix}$ A1N2

(b) attempt to find the length of $\overrightarrow{AB}$ (M1)

$\left|\overrightarrow{AB}\right| = \sqrt{6^2 + (-2)^2 + 3^2}$ $(= \sqrt{36+4+9} = \sqrt{49} = 7)$ (A1)

unit vector is $\frac{1}{7}\begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ $\left(= \begin{pmatrix} \frac{6}{7} \\ -\frac{2}{7} \\ \frac{3}{7} \end{pmatrix} \right)$ A1N2

(c) recognizing that the dot product or cos θ being 0 implies perpendicular (M1)

correct substitution in a scalar product formula A1

e.g. (6) × (–2) + (–2) × (–3) + (3) × (2), cos $\theta = \dfrac{-12+6+6}{7\times\sqrt{17}}$

correct calculation A1

e.g. $\overrightarrow{AB} \bullet \overrightarrow{AC} = 0$, cos $\theta = 0$

therefore, they are perpendicular AGN0

[8]

7.) (a) (i) (3, –4, 0) A1 N1

(ii) choosing velocity vector $\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ (M1)

finding magnitude of velocity vector (A1)

e.g. $\sqrt{(-2)^2+3^2+1^2}$, $\sqrt{4+9+1}$

speed = 3.74 ($\sqrt{14}$) A1N2

(b) (i) substituting $p = 7$ (M1)

B = (–11, 17, 7) A1 N2

(ii) **METHOD 1**

appropriate method to find $\overrightarrow{AB}$ or $\overrightarrow{BA}$ (M1)

e.g. $\overrightarrow{AO}+\overrightarrow{OB}$, A – B

$\overrightarrow{AB}=\begin{pmatrix}-14\\21\\7\end{pmatrix}$ or $\overrightarrow{BA}=\begin{pmatrix}14\\-21\\-7\end{pmatrix}$ (A1)

distance = 26.2 ($7\sqrt{14}$) A1N3

METHOD 2

evidence of applying distance is speed × time (M2)

e.g. 3.74 × 7

distance = 26.2 ($7\sqrt{14}$) A1N3

METHOD 3

attempt to find AB^2, AB (M1)

e.g. $(3-(-11))^2+(-4-17)^2+(0-7)^2$, $\sqrt{(3-(-11))^2+(-4-17)^2+(0-7)^2}$

$AB^2 = 686$, AB = $\sqrt{686}$ (A1)

distance AB = 26.2 ($7\sqrt{14}$) A1N3

(c) correct direction vectors $\begin{pmatrix}-2\\3\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\2\\a\end{pmatrix}$ (A1)(A1)

$\left|\begin{matrix}-1\\2\\a\end{matrix}\right|=\sqrt{a^2+5}$, $\begin{pmatrix}-2\\3\\1\end{pmatrix}\bullet\begin{pmatrix}-1\\2\\a\end{pmatrix}=a+8$ (A1)(A1)

substituting M1

e.g. $\cos 40° = \dfrac{a+8}{\sqrt{14}\sqrt{a^2+5}}$

$a = 3.21$, $a = -0.990$ A1A1N3

[16]

8.) (a) (i) correct approach A1

e.g. $\overrightarrow{OC}-\overrightarrow{OA}$, $\begin{pmatrix}5\\2\end{pmatrix}-\begin{pmatrix}1\\0\end{pmatrix}$

$\overrightarrow{AC}=\begin{pmatrix}4\\2\end{pmatrix}$ AG N0

(ii) appropriate approach (M1)

e.g. D – B, $\begin{pmatrix}4\\-1\end{pmatrix}-\begin{pmatrix}1\\5\end{pmatrix}$, move 3 to the right and 6 down

$\overrightarrow{BD} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$ A1N2

(iii) finding the scalar product A1
e.g. 4(3) + 2(–6), 12 – 12

valid reasoning R1
e.g. 4(3) + 2(–6) = 0, scalar product is zero
$\overrightarrow{AC}$ is perpendicular to $\overrightarrow{BD}$ AGN0

(b) (i)correct "position" vector for ***u***; "direction" vector for ***v***A1A1N2

e.g. $\boldsymbol{u} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}, \boldsymbol{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \boldsymbol{v} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \boldsymbol{v} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

accept in equation *e.g.* $\begin{pmatrix} 5 \\ 2 \end{pmatrix} + t\begin{pmatrix} -4 \\ -2 \end{pmatrix}$

(ii) any correct equation in the form ***r*** = ***a*** + *t****b***, where ***b*** = $\overrightarrow{BD}$

e.g. $\boldsymbol{r} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ -6 \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + t\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ A2N2

(c) **METHOD 1**

substitute (3, *k*) into equation for (AC) or (BD) (M1)
e.g. 3 = 1 + 4*s*, 3 = 1 + 3*t*

value of *t* or *s* A1

e.g. $s = \frac{1}{2}, -\frac{1}{2}, t = \frac{2}{3}, -\frac{1}{3},$

substituting A1

e.g. $k = 0 + \frac{1}{2}(2),$

$k = 1$ AGN0

METHOD 2

setting up two equations (M1)
e.g. 1 + 4*s* = 4 + 3*t*, 2*s* = –1 – 6*t*; setting vector equations of lines equal

value of *t* or *s* A1

e.g. $s = \frac{1}{2}, -\frac{1}{2}, t = \frac{2}{3}, -\frac{1}{3}$

substituting A1

e.g. $\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 3 \\ -6 \end{pmatrix},$

$k = 1$ AGN0

(d) $\overrightarrow{PD} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ (A1)

$\left|\overrightarrow{PD}\right| = \sqrt{2^2 + 1^2} \, (= \sqrt{5})$ (A1)

$\left|\overrightarrow{AC}\right| = \sqrt{4^2 + 2^2} \, (= \sqrt{20})$ (A1)

area = $\frac{1}{2} \times \left|\overrightarrow{AC}\right| \times \left|\overrightarrow{PD}\right| \quad \left(= \frac{1}{2} \times \sqrt{20} \times \sqrt{5}\right)$ M1

$= 5$ A1N4

[17]

9.) correct substitutions for $\boldsymbol{v} \bullet \boldsymbol{w}$; $|\boldsymbol{v}|$; $|\boldsymbol{w}|$ (A1)(A1)(A1)

e.g. $2k + (-3) \times (-2) + 6 \times 4$, $2k + 30$; $\sqrt{2^2 + (-3)^2 + 6^2}, \sqrt{49}; \sqrt{k^2 + (-2)^2 + 4^2}, \sqrt{k^2 + 20}$

evidence of substituting into the formula for scalar product (M1)

e.g. $\dfrac{2k + 30}{7 \times \sqrt{k^2 + 20}}$

correct substitution A1

e.g. $\cos\dfrac{\pi}{3} = \dfrac{2k + 30}{7 \times \sqrt{k^2 + 20}}$

$k = 18.8$ A2 N5

[7]

10.) (a) (i) evidence of approach (M1)

e.g. $\overrightarrow{\mathrm{PQ}} = \overrightarrow{\mathrm{PO}} + \overrightarrow{\mathrm{OQ}}$, Q – P

$\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ A1 N2

(ii) $\overrightarrow{\mathrm{PR}} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$ A1N1

(b) **METHOD 1**

choosing correct vectors $\overrightarrow{\mathrm{PQ}}$ and $\overrightarrow{\mathrm{PR}}$ (A1)(A1)

finding $\overrightarrow{\mathrm{PQ}} \bullet \overrightarrow{\mathrm{PR}}, \left|\overrightarrow{\mathrm{PQ}}\right|, \left|\overrightarrow{\mathrm{PR}}\right|$ (A1) (A1)(A1)

$\overrightarrow{\mathrm{PQ}} \bullet \overrightarrow{\mathrm{PR}} = -2 + 4 + 4\ (= 6)$

$\left|\overrightarrow{\mathrm{PQ}}\right| = \sqrt{(-1)^2 + 2^2 + 1^2} = (\sqrt{6}), \left|\overrightarrow{\mathrm{PR}}\right| = \sqrt{2^2 + 2^2 + 4^2}\ (= \sqrt{24})$

substituting into formula for angle between two vectors M1

e.g. $\cos \mathrm{R\hat{P}Q} = \dfrac{6}{\sqrt{6} \times \sqrt{24}}$

simplifying to expression clearly leading to $\dfrac{1}{2}$ A1

e.g. $\dfrac{6}{\sqrt{6} \times 2\sqrt{6}}, \dfrac{6}{\sqrt{144}}, \dfrac{6}{12}$

$\cos \mathrm{R\hat{P}Q} = \dfrac{1}{2}$ AGN0

METHOD 2

evidence of choosing cosine rule (seen anywhere) (M1)

$\overrightarrow{\mathrm{QR}} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$ A1

$\left|\overrightarrow{QR}\right| = \sqrt{18}, \left|\overrightarrow{PQ}\right| = \sqrt{6}$ and $\left|\overrightarrow{PR}\right| = \sqrt{24}$ (A1)(A1)(A1)

$\cos R\hat{P}Q = \dfrac{(\sqrt{6})^2 + (\sqrt{24})^2 - (\sqrt{18})^2}{2\sqrt{6} \times \sqrt{24}}$ A1

$\cos R\hat{P}Q = \dfrac{6 + 24 - 18}{24} \left(= \dfrac{12}{24} \right)$ A1

$\cos R\hat{P}Q = \dfrac{1}{2}$ AGN0

(c) (i) **METHOD 1**

evidence of appropriate approach (M1)

e.g. using $\sin^2 R\hat{P}Q + \cos^2 R\hat{P}Q = 1$, diagram

substituting correctly (A1)

e.g. $\sin R\hat{P}Q = \sqrt{1 - \left(\dfrac{1}{2}\right)^2}$

$\sin R\hat{P}Q = \sqrt{\dfrac{3}{4}} \quad \left(= \dfrac{\sqrt{3}}{2} \right)$ A1N3

METHOD 2

since $\cos \hat{P} = \dfrac{1}{2}, \hat{P} = 60°$ (A1)

evidence of approach

e.g. drawing a right triangle, finding the missing side (A1)

$\sin \hat{P} = \dfrac{\sqrt{3}}{2}$ A1N3

(ii) evidence of appropriate approach (M1)

e.g. attempt to substitute into $\dfrac{1}{2} ab \sin C$

correct substitution

e.g. area = $\dfrac{1}{2}\sqrt{6} \times \sqrt{24} \times \dfrac{\sqrt{3}}{2}$ A1

area = $3\sqrt{3}$ A1N2

[16]

11.) finding scalar product and magnitudes (A1)(A1)(A1)

scalar product = 12 – 20 – 15 (= 23)

magnitudes = $\sqrt{3^2 + 4^2 + 5^2}, \sqrt{4^2 + (-5)^2 + (-3)^2} \quad (\sqrt{50}, \sqrt{50})$

substitution into formula M1

e.g. $\cos \theta = \dfrac{12 - 20 - 15}{(\sqrt{3^2 + 4^2 + 5^2}) \times (\sqrt{4^2 + (-5)^2 + (-3)^2})}$

$\cos \theta = -\dfrac{23}{50}$ (= –0.46) A2 N4

[6]

12.) (a) L_1: $\boldsymbol{r} = = \begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ A2 N2

(b) evidence of equating $\boldsymbol{r}$ and $\overrightarrow{\mathrm{OA}}$ (M1)

e.g. $\begin{pmatrix} 6 \\ 2 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}, A = r$

one correct equation A1
e.g. $6 = 2 + 2s$, $2 = 4 - s$, $9 = -1 + 5s$, $s=2$ A1

evidence of confirming for other **two** equations A1
e.g. $6 = 2 + 4$, $2 = 4 - 2$, $9 = -1 + 10$
so A lies on L_2 AGN0

(c) (i) evidence of approach M1

e.g. $\begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $L_1 = L_2$

one correct equation A1
e.g. $2 + 2s = 8$, $4 - s = 1$, $-1 + 5s = t$

attempt to solve (M1)
finding $s = 3$ A1

substituting M1

e.g. $\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + 3\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$

$\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 8 \\ 1 \\ 14 \end{pmatrix}$ AGN0

(ii) evidence of appropriate approach (M1)
e.g. $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{AO}} + \overrightarrow{\mathrm{OB}}, \overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$ A1N2

(d) evidence of appropriate approach (M1)
e.g. $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{DC}}$

correct values A1

e.g. $\overrightarrow{\mathrm{OD}} + \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}, \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2-x \\ 1-y \\ -4-z \end{pmatrix}$

$\overrightarrow{\mathrm{OD}} = \begin{pmatrix} 0 \\ 2 \\ -9 \end{pmatrix}$ A1N2

[16]

13.) evidence of appropriate approach (M1)

e.g. $\begin{pmatrix}2\\3\\-1\end{pmatrix}+s\begin{pmatrix}5\\-3\\2\end{pmatrix}=\begin{pmatrix}9\\2\\2\end{pmatrix}+t\begin{pmatrix}-3\\5\\-1\end{pmatrix}$

two correct equations A1A1

e.g. $2 + 5s = 9 – 3t$, $3 – 3s = 2 + 5t$, $–1 + 2s = 2 – t$

attempting to solve the equations (M1)

one correct parameter $s = 2$, $t = –1$ A1

P is (12, –3, 3) $\left(\text{accept}\begin{pmatrix}12\\-3\\3\end{pmatrix}\right)$ A1 N3

[6]

14.) (a) evidence of equating scalar product to 0 (M1)

$2 \times 3 + 3 \times (–1) + (–1) \times p = 0$ $(6 – 3 – p = 0, 3 – p = 0)$ A1

$p = 3$ A1 N2

(b) evidence of substituting into magnitude formula (M1)

e.g. $\sqrt{1+q^2+25}$, $1 + q^2 + 25$

setting up a correct equation A1

e.g. $\sqrt{1+q^2+25}=\sqrt{42}$, $1 + q^2 + 25 = 42$, $q^2 = 16$

$q = \pm 4$ A1N2

[6]

15.) (a) evidence of correct approach A1

e.g. $\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}, \begin{pmatrix}3\\-3\\8\end{pmatrix}-\begin{pmatrix}2\\-1\\5\end{pmatrix}$

$\overrightarrow{PQ}=\begin{pmatrix}1\\-2\\3\end{pmatrix}$ AGN0

(b) (i) correct description R1 N1

e.g. reference to $\begin{pmatrix}3\\-3\\8\end{pmatrix}$ being the position vector of a point on the line,

a vector to the line, a point on the line.

(ii) **any** correct expression in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ A2N2

where $\boldsymbol{a}$ is $\begin{pmatrix}3\\-3\\8\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}1\\-2\\3\end{pmatrix}$

$e.g.\ \boldsymbol{r} = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t\begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}, \boldsymbol{r} = \begin{pmatrix} 3+2s \\ -3-4s \\ 8+6s \end{pmatrix}$

(c) **one** correct equation (A1)
e.g. $3 + s = -1, -3 - 2s = 5$

$s = -4$ A1
$p = -4$ A1N2

(d) **one** correct equation A1
e.g. $-3 + t = -1, 9 - 2t = 5$

$t = 2$ A1
substituting $t = 2$
e.g. $2 + 2q = -4, 2q = -6$ A1

$q = -3$ AGN0

(e) choosing correct direction vectors $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$ (A1)(A1)

finding correct scalar product and magnitudes (A1)(A1)(A1)

scalar product $(1)(1) + (-2)(-2) + (-3)(3)$ $(= -4)$
magnitudes $\sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}, \sqrt{1^2 + (-2)^3 + (-3)^2} = \sqrt{14}$
evidence of substituting into scalar product M1
e.g. $\cos\theta = \dfrac{-4}{3.741\ldots \times 3.741\ldots}$
$\theta = 1.86$ radians (or 107°) A1N4

[17]

16.) (a) (i) evidence of combining vectors (M1)

e.g. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ (or $\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$ in part (ii))

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$ A1 N2

(ii) $\overrightarrow{AD} = \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix}$ A1 N1

(b) evidence of using perpendicularity $\Rightarrow$ scalar product = 0 (M1)

e.g. $\begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix} = 0$

$4 - 4(k - 5) + 4 = 0$ A1

$-4k + 28 = 0$ (accept any correct equation clearly leading to $k = 7$) A1

$k = 7$ AG N0

(c) $\vec{AD} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$ (A1)

$\vec{BC} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ A1

evidence of correct approach (M1)

e.g. $\vec{OC} = \vec{OB} + \vec{BC}$, $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} x-3 \\ y-1 \\ z-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

$\vec{OC} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ A1 N3

(d) **METHOD 1**

choosing appropriate vectors, $\vec{BA}$, $\vec{BC}$ (A1)

finding the scalar product M1

e.g. –2(1) + 4(1) + 2(–1), 2(1) + (–4)(1) + (–2)(–1)

cos $A\hat{B}C = 0$ A1 N1

METHOD 2

$\vec{BC}$ parallel to $\vec{AD}$ (may show this on a diagram with points labelled) R1

$\vec{BC} \perp \vec{AB}$ (may show this on a diagram with points labelled) R1

$A\hat{B}C = 90°$

cos $A\hat{B}C = 0$ A1 N1

[13]

17.) $p\boldsymbol{w} = p\boldsymbol{i} + 2p\boldsymbol{j} - 3p\boldsymbol{k}$ (seen anywhere) (A1)

attempt to find $\boldsymbol{v} + p\boldsymbol{w}$ (M1)

e.g. $3\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k} + p(\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k})$

collecting terms $(3 + p)\boldsymbol{i} + (4 + 2p)\boldsymbol{j} + (1 - 3p)\,\boldsymbol{k}$ A1

attempt to find the dot product (M1)

e.g. $1(3 + p) + 2(4 + 2p) - 3(1 - 3p)$

setting **their** dot product equal to 0 (M1)

e.g. $1(3 + p) + 2(4 + 2p) - 3(1 - 3p) = 0$

simplifying A1

e.g. $3 + p + 8 + 4p - 3 + 9p = 0$, $14p + 8 = 0$

$P = -0.571 \left(-\frac{8}{14}\right)$ A1 N3

[7]

18.) (a) (i) evidence of approach M1

e.g. $\vec{AO} + \vec{OB} = \vec{AB}$, B – A

$$\vec{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$$ AG N0

(ii) for choosing **correct** vectors, ($\vec{AO}$ with $\vec{AB}$, or $\vec{OA}$ with $\vec{BA}$) (A1)(A1)

Note: *Using $\overrightarrow{AO}$ with $\overrightarrow{BA}$ will lead to* $\pi -$ *0.799. If they then say $B\hat{A}O$ = 0.799, this is a correct solution.*

calculating $\vec{AO} \bullet \vec{AB}$, $\left|\vec{AO}\right|$, $\left|\vec{AB}\right|$ (A1)(A1)(A1)

e.g. $\boldsymbol{d}_1 \bullet \boldsymbol{d}_2 = (-1)(-4) + (2)(6) + (-3)(-1)$ (= 19)

$|\boldsymbol{d}_1| = \sqrt{(-1)^2 + 2^2 + (-3)^2} \left(= \sqrt{14}\right)$,

$|\boldsymbol{d}_2| = \sqrt{(-4)^2 + 6^2 + (-1)^2} \left(= \sqrt{53}\right)$

evidence of using the formula to find the angle M1

e.g. $\cos\theta = \dfrac{(-1)(-4)+(2)(6)+(-3)(-1)}{\sqrt{(-1)^2+2^2+(-3)^2}\sqrt{(-4)^2+6^2+(-1)^2}}$,

$\dfrac{19}{\sqrt{14}\sqrt{53}}$, 0.69751...

$B\hat{A}O$ = 0.799 radians (accept 45.8°) A1 N3

(b) two correct answers A1A1

e.g. (1, – 2, 3), (–3, 4, 2), (–7, 10, 1), (–11, 16, 0) N2

(c) (i) $\boldsymbol{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ A2 N2

(ii) C on L_2, so $\begin{pmatrix} k \\ -k \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ (M1)

evidence of equating components (A1)

e.g. $1 - 3t = k$, $-2 + 4t = -k$, $5 = 3 + 2t$

one correct value $t = 1$, $k = -2$ (seen anywhere) (A1)

coordinates of C are (–2, 2, 5) A1 N3

(d) for setting up one (or more) correct equation using

$$\begin{pmatrix}-2\\2\\5\end{pmatrix}=\begin{pmatrix}3\\-8\\0\end{pmatrix}+p\begin{pmatrix}1\\-2\\-1\end{pmatrix}$$ (M1)

e.g. $3 + p = -2$, $-8 - 2p = 2$, $-p = 5$

$p = -5$ A1 N2

[18]

19.) evidence of equating vectors (M1)
e.g. $L_1 = L_2$

for any **two** correct equations A1A1
e.g. $2 + s = 3 - t$, $5 + 2s = -3 + 3t$, $3 + 3s = 8 - 4t$

attempting to solve the equations (M1)
finding **one** correct parameter ($s = -1$, $t = 2$) A1

the coordinates of T are (1, 3, 0) A1 N3

[6]

20.) (a) (i) evidence of approach (M1)

e.g. $\overrightarrow{AO}+\overrightarrow{OB}$, B – A, $\begin{pmatrix}9-6\\-6+2\\15-10\end{pmatrix}$

$\overrightarrow{AB}=\begin{pmatrix}3\\-4\\5\end{pmatrix}$ (accept (3, 4, 5)) A1 N2

(ii) evidence of finding the magnitude of the velocity vector M1
e.g. speed = $\sqrt{3^2+4^2+5^2}$
speed = $\sqrt{50}$ $(=5\sqrt{2})$ A1 N1

(b) correct **equation** (accept Cartesian and parametric forms) A2 N2

e.g. $\boldsymbol{r}=\begin{pmatrix}6\\-2\\10\end{pmatrix}+t\begin{pmatrix}3\\-4\\5\end{pmatrix}$, $\boldsymbol{r}=\begin{pmatrix}9\\-6\\15\end{pmatrix}+t\begin{pmatrix}3\\-4\\5\end{pmatrix}$

[6]

21.) (a) (i) evidence of approach M1

e.g. B – A, $\overrightarrow{AO}+\overrightarrow{OB}$, $\begin{pmatrix}6\\4\\4\end{pmatrix}-\begin{pmatrix}1\\2\\3\end{pmatrix}$

$\overrightarrow{AB}=\begin{pmatrix}5\\2\\1\end{pmatrix}$ AG N0

(ii) evidence of approach (M1)

e.g. $D - A$, $\overrightarrow{AO} + \overrightarrow{OD}$, $\begin{pmatrix} 2 \\ 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$\overrightarrow{AD} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ A1 N2

(iii) evidence of approach (M1)

e.g. $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD}$

correct substitution A1

e.g. $\overrightarrow{AC} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix}$ AG N0

(b) evidence of combining vectors (there are at least 5 ways) (M1)

e.g. $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$, $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{AD}$, $\overrightarrow{AB} = \overrightarrow{OC} - \overrightarrow{OD}$

correct substitution A1

e.g. $\overrightarrow{OC} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix} \quad \left(= \begin{pmatrix} 7 \\ 7 \\ 6 \end{pmatrix} \right)$

coordinates of C are (7, 7, 6) A1 N1

(c) (i) evidence of using scalar product on $\overrightarrow{AB}$ and $\overrightarrow{AD}$ (M1)

e.g. $\overrightarrow{AB} \bullet \overrightarrow{AD} = 5(1) + 2(3) + 1(2)$

$\overrightarrow{AB} \bullet \overrightarrow{AD} = 13$ A1 N2

(ii) $\left|\overrightarrow{AB}\right| = 5.477...$, $\left|\overrightarrow{AD}\right| = 3.741...$ (A1)(A1)

evidence of using $\cos A = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{AD}}{\left|\overrightarrow{AB}\right|\left|\overrightarrow{AD}\right|}$ (M1)

correct substitution A1

e.g. $\cos A = \dfrac{13}{20.493...}$

$\hat{A} = 0.884$ (50.6°) A1 N3

(d) **METHOD 1**

evidence of using area $= 2\left(\frac{1}{2}\left|\overrightarrow{AD}\right|\left|\overrightarrow{AB}\right| \sin D\hat{A}B\right)$ (M1)

correct substitution A1

e.g. area $= 2\left(\frac{1}{2}(3.741...)(5.477...)\sin 0.883...\right)$

area = 15.8 A1 N2

METHOD 2

evidence of using area = $b \times h$ (M1)

finding height of parallelogram A1

e.g. $h = 3.741... \times \sin 0.883...$ (= 2.892...), $h = 5.477... \times \sin 0.883...$ (= 4.234...)
area = 15.8 A1 N2

[18]

22.) (a) (i) evidence of combining vectors (M1)
e.g. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$ A1 N2

(ii) $\overrightarrow{AC} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ A1N1

(b) (i) $\overrightarrow{AB} \bullet \overrightarrow{AC} = (-2)(3) + (-3)(-2) = 0$ A1 N1

(ii) scalar product 0 = $\Rightarrow$ perpendicular, $\theta = 90°$ (R1)
$\sin \theta = 1$ A1N2

[6]

23.) (a) Using direction vectors $\boldsymbol{u} = \begin{pmatrix} -2 \\ 6 \\ 10 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} -6 \\ 10 \\ -2 \end{pmatrix}$ (M1)

$|\boldsymbol{u}| = \sqrt{4+36+100} = \sqrt{140}, |\boldsymbol{v}| = \sqrt{36+100+4} = \sqrt{140}$ A1A1
$\boldsymbol{u} \bullet \boldsymbol{v} = 12 + 60 - 20 = 52$ A1

$\cos \theta = \dfrac{52}{\sqrt{140}\sqrt{140}}$ A1

$= \dfrac{52}{140}$ AG N0

(b) (i) For substituting $s = 1$ (M1)
Correct calculations (A1)
$9 + 1(-2) = 7, 4 + 1(6) = 10, -6 + 1(10) = 4$

position vector of P is $\begin{pmatrix} 7 \\ 10 \\ 4 \end{pmatrix}$ A1 N3

(ii) For substituting into the equation $\begin{pmatrix} 7 \\ 10 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 20 \\ 2 \end{pmatrix} + t\begin{pmatrix} -6 \\ 10 \\ -2 \end{pmatrix}$ (M1)

For one correct equation A1
e.g. $7 = 1 - 6t$
Solving gives $t = -1$ A1

verify for second coordinate, $10 = 20 + (-1)(10)$ A1
verify for third coordinate, $4 = 2 + (-1)(-2)$ A1
Thus, P is also on L_2. AGN0

(c) $k\begin{pmatrix} -2 \\ 6 \\ 10 \end{pmatrix} = \begin{pmatrix} 6 \\ x \\ -30 \end{pmatrix}$ (M1)

$-2k = 6$

$k = -3$ A1

$x = -3 \times 6 = -18$ A1N2

[16]

24.) (a) Finding **correct** vectors, $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ A1A1

Substituting correctly in the scalar product $\overrightarrow{AB} \cdot \overrightarrow{AC} = 4(-3) + 3(1)$ A1

$= -9$ AG N0

(b) $\left|\overrightarrow{AB}\right| = 5$ $\left|\overrightarrow{AC}\right| = \sqrt{10}$ (A1)(A1)

Evidence of using scalar product formula M1

e.g. $\cos B\hat{A}C = \dfrac{-9}{5\sqrt{10}}$ $= -0.569$ (3 s.f.)

$B\hat{A}C = 2.47$ (radians), 125° A1N3

[7]

25.) (a) Attempting to find unit vector ($\boldsymbol{e_b}$) in the direction of $\boldsymbol{b}$ (M1)

Correct values = $\dfrac{1}{\sqrt{3^2 + 4^2 + 0^2}} \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ A1

$= \begin{pmatrix} 0.6 \\ 0.8 \\ 0 \end{pmatrix}$ A1

Finding direction vector for $\boldsymbol{b}$, $\boldsymbol{v_b} = 18 \times \boldsymbol{e_b}$ (M1)

$\boldsymbol{b} = \begin{pmatrix} 10.8 \\ 14.4 \\ 0 \end{pmatrix}$ A1

Using vector representation $\boldsymbol{b} = \boldsymbol{b}_0 + t\boldsymbol{v_b}$ (M1)

$= \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + t\begin{pmatrix} 10.8 \\ 14.4 \\ 0 \end{pmatrix}$ AG N0

(b) (i) $t = 0 \Rightarrow (49, 32, 0)$ A1 N1

(ii) Finding magnitude of velocity vector (M1)

Substituting correctly $v_h = \sqrt{(-48)^2 + (-24)^2 + 6^2}$ A1

$= 54$ (km h^{-1}) A1N2

(c) (i) At R, $\begin{pmatrix} 10.8t \\ 14.4t \\ 5 \end{pmatrix} = \begin{pmatrix} 49 - 48t \\ 32 - 24t \\ 6t \end{pmatrix}$ A1

$t = \dfrac{5}{6}$ (= 0.833)(hours) A1 N1

(ii) For substituting $t = \dfrac{5}{6}$ into expression for $\boldsymbol{b}$ or $\boldsymbol{h}$ (M1)

(9, 12, 5) A2N3

[15]

26.) (a) $\boldsymbol{u} \bullet \boldsymbol{v} = 8 + 3 + p$ (A1)

For equating scalar product equal to zero (M1)

$8 + 3 + p = 0$

$p = -11$ A1 N3

(b) $|\boldsymbol{u}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}$, 3.74 (M1)

$q\sqrt{14} = 14$ A1

$q = \sqrt{14}$ $(= 3.74)$ A1 N2

[6]

27.) ***Note**: In this question, accept any correct vector notation, including row vectors eg (1, - 2, 3).*

(a) (i) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ (M1)

$= \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ A1 N2

(ii) $\boldsymbol{r} = \overrightarrow{OP} + s\overrightarrow{PQ}$ (M1)

$= -5\boldsymbol{i} + 11\boldsymbol{j} - 8\boldsymbol{k} + s(\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k})$ A1

$= (-5 + s)\,\boldsymbol{i} + (11 - 2s)\boldsymbol{j} + (-8 + 3s)\,\boldsymbol{k}$ AG N0

(b) If $(2, y_1, z_1)$ lies on L_1 then $-5 + s = 2$ (M1)

$s = 7$ A1

$y_1 = -3$, $z_1 = 13$ A1A1 N3

(c) Evidence of correct approach (M1)

eg $(-5 + s)\boldsymbol{i} + (11 - 2s)\,\boldsymbol{j} + (-8 + 3s)\,\boldsymbol{k} = 2\boldsymbol{i} + 9\boldsymbol{j} + 13\boldsymbol{k} + t(\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k})$

At least two correct equations A1A1

eg $-5 + s = 2 + t$, $11 - 2s = 9 + 2t$, $-8 + 3s = 13 + 3t$

Attempting to solve **their** equations (M1)

One correct parameter ($s = 4$, $t = -3$) A1

$\overrightarrow{OT} = -\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}$ A2 N4

(d) Direction vector for L_1 is $\boldsymbol{d}_1 = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ (A1)

***Note**: Award A1FT for their vector from (a)(i).*

Direction vector for L_2 is $\boldsymbol{d}_2 = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ (A1)

$\boldsymbol{d}_1 \bullet \boldsymbol{d}_2 = 6$, $|\mathrm{d}_1| = \sqrt{14}$, $|\mathrm{d}_2| = \sqrt{14}$, (A1)(A1)(A1)

$\cos\theta = \frac{6}{\sqrt{14}\sqrt{14}}$ $\left(= \frac{6}{14} = \frac{3}{7}\right)$ A1

q = 64.6° (= 1.13radians) A1 N4

***Note**: Award marks as per the markscheme if their (correct) direction vectors give*

$d_1 \bullet d_2$ = - 6, leading to q *= 115î*
(= 2.01 radians).

[22]

28.) (a) speed = $\sqrt{3^2+4^2+10^2}$ (M1)

= $\sqrt{125}$ = $5\sqrt{5}$, 11.2, (metres per minute) A1 N2

(b) Let the velocity vector be $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$

Finding a velocity vector A2

$eg \begin{pmatrix} 3 \\ 16 \\ 39 \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + 2\begin{pmatrix} a \\ b \\ c \end{pmatrix}, \begin{pmatrix} 3 \\ 16 \\ 39 \end{pmatrix} - \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix}$

Dividing by 2 to give $\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$ A1

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$ AG N0

(c) (i) At Q, $\begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$ (M1)

Setting up one correct equation A1

eg $3 + 3t = -5 + 4t$, $2 + 4t = 10 + 3t$, $7 + 10t = 23 + 8t$

$t = 8$ (A1)

Correct answer A1

eg after 8 minutes, 13:08 N3

(ii) Substituting for t (M1)

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + 8\begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}$, or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + 8\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$

$x = 27$, $y = 34$, $z = 87$ or (27, 34, 87), or $\begin{pmatrix} 27 \\ 34 \\ 87 \end{pmatrix}$ A1 N2

(d) For choosing **both** direction vectors $\boldsymbol{d}_1 = \begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}$ and $\boldsymbol{d}_2 = \begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$ (A1)

$\boldsymbol{d}_1 \bullet \boldsymbol{d}_2 = 104,\ |d_1| = \sqrt{125},\ |d_2| = \sqrt{89}$ (A1)(A1)(A1)

$\cos\theta = \dfrac{104}{\sqrt{125}\sqrt{89}} = 0.98601...$ A1

$\theta = 0.167$ (radians) (accept $\theta = 9.59°$) A1 N3

[17]

29.) (a) (i) Evidence of approach

eg $\vec{JQ} = \begin{pmatrix} 0 \\ 7 \\ 10 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix},\ \vec{JQ} = \vec{JO} + \vec{OQ}$ M1

$\vec{JQ} = \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}$ AG N0

(ii) $\vec{MK} = \begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix}$ A1 N1

(b) (i) $\boldsymbol{r} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}$ or $\boldsymbol{r} = \begin{pmatrix} 0 \\ 7 \\ 10 \end{pmatrix} + t\begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}$ A2 N2

Note: *Award A1 if "$\boldsymbol{r}$ = " is missing.*

(ii) Evidence of choosing correct vectors $\begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}, \begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix}$ (A1)(A1)

Evidence of calculating magnitudes (A1)(A1)

eg $\sqrt{(-6)^2 + 7^2 + 10^2} = \sqrt{185}\ \sqrt{6^2 + (-7)^2 + 10^2}$
$= \sqrt{185}$

$\begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix} \bullet \begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix} = -36 - 49 + 100$ (= 15) (accept − 15) (A1)

For evidence of substitution into the correct formula M1

eg $\cos\theta = \dfrac{15}{\sqrt{185}\sqrt{185}}\ \left(= \dfrac{15}{185} = 0.0811\right)$

$\left(\text{accept } \dfrac{-15}{\sqrt{185}\sqrt{185}}\right)$

$\theta = 1.49$ (radians), $85.3°$ A1 N4

(c) **METHOD 1**

Geometric approach (M1)

Valid reasoning A2

eg diagonals bisect each other, $\overrightarrow{OD} = \overrightarrow{OM} + \frac{1}{2}\overrightarrow{MK}$

Calculation of mid point (A1)

eg $\left(\frac{6+0}{2}, \frac{0+7}{2}, \frac{0+10}{2}\right)$

$\overrightarrow{OD} = \begin{pmatrix} 3 \\ 3.5 \\ 5 \end{pmatrix}$ (accept (3,3.5,5)) A1 N3

METHOD 2

Correct approach (M1)

eg $\begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} + \boldsymbol{t}\begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \\ 10 \end{pmatrix} + \boldsymbol{s}\begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix}$

Two correct equations A1

eg $6 - 6t = 6s, 7t = 7 - 7s, 10t = 10s$

Attempt to solve (M1)

One correct parameter

$s = 0.5 \quad t = 0.5$ A1

$\overrightarrow{OD} = \begin{pmatrix} 3 \\ 3.5 \\ 5 \end{pmatrix}$ (accept (3, 3.5, 5)) A1 N3

METHOD 3

Correct approach (M1)

eg $\begin{pmatrix} 0 \\ 7 \\ 10 \end{pmatrix} + t\begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + s\begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix}$

Two correct equations A1

eg $-6t = 6s$, $7 + 7t = 7 - 7s$, $10 + 10t = 10s$

Attempt to solve (M1)

One correct parameter

$s = 0.5 \quad t = -0.5$ A1

$\overrightarrow{OD} = \begin{pmatrix} 3 \\ 3.5 \\ 5 \end{pmatrix}$ (accept (3, 3.5, 5)) A1 N3

[16]

30.) (a) $\overrightarrow{PQ} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ A1A1 N2

(b) Using $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix} + t\begin{pmatrix} 5 \\ -3 \end{pmatrix}$$ A2A1A1 N4

[6]

31.) (a) (i) Evidence of subtracting all three components in the correct order M1

eg $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (4\boldsymbol{i} - 5\boldsymbol{j} + 21\boldsymbol{k}) - (2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k})$

$= 2\boldsymbol{i} - 8\boldsymbol{j} + 20\boldsymbol{k}$ AG N0

(ii) $|\overrightarrow{AB}| = \sqrt{2^2 + (-8)^2 + 20^2}$ $\left(= \sqrt{468} = 6\sqrt{13} = 2\sqrt{117} = 21.6\right)$ (A1)

$\boldsymbol{u} = \dfrac{1}{\sqrt{468}}(2\boldsymbol{i} - 8\boldsymbol{j} + 20\boldsymbol{k})$ A1 N2

$\left(= \dfrac{2}{\sqrt{468}}\boldsymbol{i} - \dfrac{8}{\sqrt{468}}\boldsymbol{j} + \dfrac{20}{\sqrt{468}}\boldsymbol{k}, 0.0925\boldsymbol{i} - 0.370\boldsymbol{j} + 0.925\boldsymbol{k}, \text{etc.}\right)$

(iii) If the scalar product is zero, the vectors are perpendicular. R1

***Note**: Award R1 for stating the relationship between the scalar product and perpendicularity, seen anywhere in the solution.*

Finding an appropriate scalar product $\left(\boldsymbol{u} \bullet \overrightarrow{OA} \text{ or } \overrightarrow{AB} \bullet \overrightarrow{OA}\right)$ M1

eg $\boldsymbol{u} \bullet \overrightarrow{OA} = \left(\dfrac{2}{\sqrt{468}}\right) \times 2 + \left(\dfrac{-8}{\sqrt{468}}\right) \times 3 + \left(\dfrac{20}{\sqrt{468}}\right) \times 1$

$\left(= \dfrac{4 - 24 + 20}{\sqrt{468}}\right)$

$\overrightarrow{AB} \bullet \overrightarrow{OA} = 2 \times 2 + (-8) \times 3 + 20 \times 1$

$\boldsymbol{u} \bullet \overrightarrow{OA} = 0$ or $\overrightarrow{AB} \bullet \overrightarrow{OA} = 0$ A1 N0

(b) (i) **EITHER**

$S\left(\dfrac{2+4}{2}, \dfrac{3-5}{2}, \dfrac{1+21}{2}\right)$ (M1)(A1)

Therefore, $\overrightarrow{OS} = 3\boldsymbol{i} - \boldsymbol{j} + 11\boldsymbol{k}$ (accept (3, –1, 11)) A1 N3

OR

$\overrightarrow{OS} = \overrightarrow{OA} + \dfrac{1}{2}\overrightarrow{AB}$ (M1)

$= (2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}) + \dfrac{1}{2}(2\boldsymbol{i} + 8\boldsymbol{j} + 20\boldsymbol{k})$ (A1)

$\overrightarrow{OS} = 3\boldsymbol{i} - \boldsymbol{j} + 11\boldsymbol{k}$ A1 N3

(ii) $L_1 : r = (3\boldsymbol{i} - \boldsymbol{j} + 11\boldsymbol{k}) + t(2\boldsymbol{i} + 3\boldsymbol{j} + 1\boldsymbol{k})$ A1 N1

(c) Using direction vectors (*eg* $2\boldsymbol{i} + 3\boldsymbol{j} + 1\boldsymbol{k}$ and $-2\boldsymbol{i} + 5\boldsymbol{j} - 3\boldsymbol{k}$) (M1)
Valid explanation of why L_1 is not parallel to L_2 R1 N2

eg. Direction vectors are not scalar multiples of each other.
Angle between the direction vectors is not zero or 180.
Finding the angle

$\boldsymbol{d}_1 \bullet \boldsymbol{d}_2 \neq |\boldsymbol{d}_1||\boldsymbol{d}_2|$.

***Note**: Award R0 for "direction vectors are not equal".*

(d) Setting up any **two** of the three equations (M1)
For each correct equation A1A1
eg $3 + 2t = 5 - 2s$, $-1 + 3t = 10 + 5s$, $11 + t = 10 - 3s$

Attempt to solve these equations (M1)
Finding **one** correct parameter ($s = -1$, $t = 2$) (A1)

P has position vector $7\boldsymbol{i} + 5\boldsymbol{j} + 13\boldsymbol{k}$ A2 N4

***Notes**: Award (M1)A2 if the same parameter is used for both lines in the initial correct equations. Award no further marks.*

[19]

32.) (a) (i) $\vec{AB} = \vec{OB} - \vec{OA}$ (A1)

$$= \begin{pmatrix} 17 \\ 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 7 \\ -3 \\ -5 \end{pmatrix}$$

$$= \begin{pmatrix} 10 \\ 5 \\ 10 \end{pmatrix}$$ A1 N2

(ii) $\vec{AB} = \sqrt{10^2 + 5^2 + 10^2}$ (M1)

$= 15$ A1 N2

(b) Evidence of correct calculation of scalar product (may be in (i), (ii) or (iii)) A1

(i) $\vec{AB} \cdot \vec{AE} = 0$ $((-6)(-2) + 6(-4) + 3(4))$ A1 N1

(ii) $\vec{AB} \cdot \vec{AD} = 0$ $((10)(-6) + 5(6) + 10(3))$ A1 N1

(iii) $\vec{AB} \cdot \vec{AE} = 0$ $((10)(-2) + 5(-4) + 10(4))$ A1 N1

(iv) $90°$ $\left(\text{or } \frac{\pi}{2}\right)$ A1 N1

(c) Volume $= |\vec{AB}| \times |\vec{AD}| \times |\vec{AE}|$ (A1)

$= 15 \times 9 \times 6$

$= 810$ (cubic units) A1 N2

(d) Setting up a valid equation involving H. There are many possibilities.

eg $\vec{OH} = \vec{OG} + \vec{GH}, \vec{OH} = \vec{OA} + \vec{AE} + \vec{EH}, \begin{pmatrix} x-9 \\ y-4 \\ z-12 \end{pmatrix} = \begin{pmatrix} -10 \\ -5 \\ -10 \end{pmatrix}$ (M1)

Using equal vectors (M1)

eg $\vec{GH} = -\vec{AB}, \vec{EH} = \vec{AD}$

$$\vec{OH} = \begin{pmatrix} 9 \\ 4 \\ 12 \end{pmatrix} - \begin{pmatrix} 10 \\ 5 \\ 10 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}, \vec{OH} = \begin{pmatrix} 7 \\ -3 \\ -5 \end{pmatrix} + \begin{pmatrix} -2 \\ -4 \\ 4 \end{pmatrix} + \begin{pmatrix} -6 \\ 6 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$$

coordinates of H are (− 1, −1, 2) A1 N3

(e) $\vec{HB} = \begin{pmatrix} 18 \\ 3 \\ 3 \end{pmatrix}$ A1

Attempting to use formula cos $\hat{P} = \dfrac{\vec{AG} \cdot \vec{HB}}{\left|\vec{AG}\right|\left|\vec{HB}\right|}$ (M1)

$= \dfrac{2\times18+7\times3+17\times3}{\sqrt{2^2+7^2+17^2}\sqrt{18^2+3^2+3^2}} \left(= \dfrac{108}{\sqrt{342}\sqrt{342}}\right)$ A1

= 0.31578... (A1)

$\hat{P} = 71.6°$ (= 1.25 radians) A1 N3

[19]

33.) (a) $\vec{AB} = \vec{OB} - \vec{OA} \left(= \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \right)$ (M1)

$\vec{AB} = \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$ A2 N3

(b) Using ***r*** = ***a*** + *t**b***

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} + t\begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$ A1A1A1 N3

[6]

34.) (a) $\vec{AB} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}, \vec{OR} = \begin{pmatrix} x \\ 3-3x \end{pmatrix}$ A1A1 N2

(b) $\vec{AB}\bullet\vec{OR}=x-3(3-3x)$ A1

$\vec{AB}\bullet\vec{OR}=0 \quad (10x-9=0)$ M1

R is $\left(\frac{9}{10},\frac{3}{10}\right)$ A1A1 N2

[6]

35.) (a) $\overrightarrow{OG} = 5\boldsymbol{i} + 5\boldsymbol{j} - 5\boldsymbol{k}$ A2 2

(b) $\overrightarrow{BD} = 5\boldsymbol{i} + 5\boldsymbol{k}$ A2 2

(c) $\overrightarrow{EB} = 5\boldsymbol{i} + 5\boldsymbol{j} - 5\boldsymbol{k}$ A2 2

***Note**: Award A0(A2)(A2) if the 5 is consistently omitted.*

[6]

36.) (a) Finding correct vectors, $\overrightarrow{AB} = \begin{pmatrix}4\\3\end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix}-3\\1\end{pmatrix}$ A1A1

Substituting correctly in the scalar product

$\overrightarrow{AB}\cdot\overrightarrow{AC} = 4(-3) + 3(1)$ A1

$= -9$ AG 3

(b) $|\overrightarrow{AB}| = 5$ $|\overrightarrow{AC}| = \sqrt{10}$ (A1)(A1)

Attempting to use scalar product formula cos BAC = $\frac{-9}{5\sqrt{10}}$ M1

= –0.569 (3 s.f) AG 3

[6]

37.) (a) Attempting to find unit vector (e_b) in the direction of $\boldsymbol{b}$ (M1)

Correct values = $\frac{1}{\sqrt{3^2+4^2+0^2}}\begin{pmatrix}3\\4\\0\end{pmatrix}$ A1

$= \begin{pmatrix}0.6\\0.8\\0\end{pmatrix}$ A1

Finding direction vector for $\boldsymbol{b}$, $\boldsymbol{v_b} = 18 \times \boldsymbol{e_b}$ (M1)

$\boldsymbol{b} = \begin{pmatrix}10.8\\14.4\\0\end{pmatrix}$ A1

Using vector representation $\boldsymbol{b} = \boldsymbol{b}_0 + t\boldsymbol{v_b}$ (M1)

$= \begin{pmatrix}0\\0\\5\end{pmatrix} + t\begin{pmatrix}10.8\\14.4\\0\end{pmatrix}$ AG 6

(b) (i) $t = 0 \Rightarrow (49, 32, 0)$ A1 1

(ii) Finding magnitude of velocity vector (M1)

Substituting correctly $v_h = \sqrt{(-48)^2 + (-24)^2 + 6^2}$ A1

$= 54(\text{km h}^{-1})$ A1 3

(c) (i) At R, $\begin{pmatrix} 10.8t \\ 14.4t \\ 5 \end{pmatrix} = \begin{pmatrix} 49 - 48t \\ 32 - 24t \\ 6t \end{pmatrix}$ A1

$t = \frac{5}{6}$ (= 0.833) (hours) A1 2

(ii) For substituting $t = \frac{5}{6}$ into expression for b or h M1

(9,12,5) A2 3

[15]

38.) **METHOD 1**

Using $\boldsymbol{a} \cdot \boldsymbol{b} = ab \cos \theta$ (may be implied) (M1)

$\begin{pmatrix} 3 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \left|\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right| \left|\begin{pmatrix} -2 \\ 1 \end{pmatrix}\right| \cos -$ (A1)

Correct value of scalar product $\begin{pmatrix} 3 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ 1 \end{pmatrix} = (3 \times -2) + (4 \times 1) = -2$ (A1)

Correct magnitudes $\left|\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right| = \sqrt{25} \, (= 5)$, $\left|\begin{pmatrix} -2 \\ 1 \end{pmatrix}\right| = \sqrt{5}$ (A1)(A1)

$\cos\theta = \frac{-2}{\sqrt{125}}$ (A1) (C6)

METHOD 2

$\left|\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right| = \sqrt{25}$ (A1)

$\left|\begin{pmatrix} -2 \\ 1 \end{pmatrix}\right| = \sqrt{5}$ (A1)

$\left|\begin{pmatrix} 5 \\ 3 \end{pmatrix}\right| = \sqrt{34}$ (A1)

Using cosine rule (M1)

$34 = 25 + 5 - 25\sqrt{5}\cos\theta$ (A1)

$\cos\theta = -\frac{2}{\sqrt{125}}$ (A1) (C6)

[6]

39.) (a) (i) $\overrightarrow{AB} = \begin{pmatrix} 200 \\ 400 \end{pmatrix} - \begin{pmatrix} -600 \\ -200 \end{pmatrix}$ (A1)

$= \begin{pmatrix} 800 \\ 600 \end{pmatrix}$ (A1) (N2)

(ii) $|\overrightarrow{AB}| = \sqrt{800^2 + 600^2} = 1000$ (must be seen) (M1)

unit vector $= \frac{1}{1000}\begin{pmatrix} 800 \\ 600 \end{pmatrix}$ (A1)

$= \begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix}$ (AG) (N0)4

Note: *A reverse method is not acceptable in "show that" questions.*

(b) (i) $\boldsymbol{v} = 250\begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix}$ (M1)

$= \begin{pmatrix} 200 \\ 150 \end{pmatrix}$ (AG) (N0)

Note: *A correct alternative method is using the given vector equation with $t = 4$.*

(ii) at 13:00, $t = 1$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -600 \\ -200 \end{pmatrix} + 1\begin{pmatrix} 200 \\ 150 \end{pmatrix}$ (M1)

$= \begin{pmatrix} -400 \\ -50 \end{pmatrix}$ (A1) (N1)

(iii) $|\overrightarrow{AB}| = 1\,000$

Time $= \frac{1\,000}{250} = 4$ (hours) (M1)(A1)

over town B at 16:00 (4 pm, 4:00 pm)
(Do not accept 16 or 4:00 or 4) (A1) (N3)6

(c) ***Note:*** *There are a variety of approaches. The table shows some of them, with the mark allocation. Use discretion, following this allocation as closely as possible.*

Time for A to B to C = 9 hours	Distance from A to B to C = 2250 km	Fuel used from A to B $= 1800 \times 4 = 7200$ litres	(A1)
Light goes on after 16000 litres	Light goes on after 16000 litres	Fuel remaining = 9800 litres	(A1)
Time for 16 000 litres	Distance on 16000 litres $= \frac{16000}{1800} \times 250$	Hours before light $\frac{8800}{1800}$ $= 4\frac{8}{9} (= 4.889)$	(A1) (A1)

$=\frac{16000}{1800}$ $=8\frac{8}{9}(=8.889)$ Time remaining is $=\frac{1}{9}(=0.111)$ hour	$=2222\frac{2}{9}(=2222.22)$ km	Time remaining is $=\frac{1}{9}(=0.111)$ hour	(A1)
Distance $=\frac{1}{9}\times 250$ = 27.8 km	Distance to C = 2250 – 2222.22 = 27.8 km	Distance $=\frac{1}{9}\times 250$ = 27.8 km	(A2) (N4) 7

[17]

40.) (a) $\sqrt{16+9} = \sqrt{25} = 5$ (M1)(A1) (C2)

(b) $\begin{pmatrix}-2\\1\end{pmatrix}+2\begin{pmatrix}4\\3\end{pmatrix}=\begin{pmatrix}6\\7\end{pmatrix}$ (so B is (6, 7)) (M1)(A1) (C2)

(c) $\boldsymbol{r} = \begin{pmatrix}-2\\1\end{pmatrix}+t\begin{pmatrix}4\\3\end{pmatrix}$ (not unique) (A2) (C2)

Note: *Award (A1) if “ **r** = ” is omitted, ie not an equation.*

[6]

41.) (a) $\overrightarrow{\text{DE}} = \begin{pmatrix}12-4\\11-5\end{pmatrix} = \begin{pmatrix}8\\6\end{pmatrix}$ (M1)(A1) (N2)

(b) $|\overrightarrow{\text{DE}}| = \sqrt{8^2+6^2}$ $\left(=\sqrt{64+36}\right)$ (M1)

= 10 (A1) (N2)

(c) **Vector geometry approach**

Using DG = 10 (M1)

$(x-4)^2 + (y-5)^2 = 100$ (A1)

Using (DG) perpendicular to (DE) (M1)

Leading to $\overrightarrow{\text{DG}} = \begin{pmatrix}-6\\8\end{pmatrix}$, $\overrightarrow{\text{DG}} = \begin{pmatrix}6\\-8\end{pmatrix}$ (A1)(A1)

Using $\overrightarrow{\text{DG}} = \overrightarrow{\text{DO}}+\overrightarrow{\text{OG}}$ (O is the origin) (M1)

G (–2, 13), G (10, –3) (accept position vectors) (A1)(A1)

Algebraic approach

gradient of DE = $\frac{6}{8}$ (A1)

gradient of DG = $-\frac{8}{6}$ (A1)

equation of line DG is $y - 5 = -\frac{4}{3}(x - 4)$ (A1)

Using DG = 10 (M1)

$(x - 4)^2 + (y - 5)^2 = 100$ (A1)

Solving simultaneous equation (M1)

G (– 2, 13), G (10, –3) (accept position vectors) (A1)(A1)

***Note**: Award full marks for an appropriately labelled diagram (eg showing that DG =10 , displacements of 6 and 8), or an **accurate** diagram leading to the correct answers.*

[12]

42.) (a) $p = 2 \Rightarrow \begin{pmatrix} 0 \\ 12 \end{pmatrix} + 2\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ (A1)

$= \begin{pmatrix} 10 \\ 6 \end{pmatrix}$ (accept any other vector notation, including (10, 6)) (A1) (N2)

(b) **METHOD 1**

(i) equating components (M1)

$0 + 5p = 14 + q$, $12 - 3p = 0 + 3q$ (A1)

$p = 3$, $q = 1$ (A1)(A1)(N1)(N1)

(ii) The coordinates of P are (15, 3) (accept $x = 15$, $y = 3$) (A1)(A1)(N1)(N1)

METHOD 2

(i) Setting up Cartesian equations (M1)

$x = 5p$ $x = 14 + q$

$y = 12 - 3p$ $y = 3q$

giving $3x + 5y = 60$ $3x - y = 42$ (A1)

Solving simultaneously gives $x = 15$, $y = 3$

Substituting to find p and q

$\begin{pmatrix} 15 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 12 \end{pmatrix} + \boldsymbol{p}\begin{pmatrix} 5 \\ -3 \end{pmatrix}, \begin{pmatrix} 15 \\ 3 \end{pmatrix} = \begin{pmatrix} 14 \\ 0 \end{pmatrix} + \boldsymbol{q}\begin{pmatrix} 1 \\ 3 \end{pmatrix},$

$p = 3$ $q = 1$ (A1)(A1)(N1)(N1)

(ii) From above, P is (15, 3) (accept $x = 15$, $y = 3$ seen above) (A1)(A1)(N1)(N1)

[8]

43.) Direction vectors are $\boldsymbol{a} = \boldsymbol{i} - 3\boldsymbol{j}$ and $\boldsymbol{b} = \boldsymbol{i} - \boldsymbol{j}$. (A2)

$\boldsymbol{a} \cdot \boldsymbol{b} = (1 + 3)$ (A1)

$|\boldsymbol{a}| = \sqrt{10}$, $|\boldsymbol{b}| = \sqrt{2}$ (A1)

$\cos\theta = \frac{\boldsymbol{a} \cdot \boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}\left(= \frac{4}{\sqrt{10}\sqrt{2}}\right)$ (M1)

$\cos\theta = \frac{4}{\sqrt{20}}$ (A1) (C6)

[6]

44.) (a) (i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ (M1)

$= \begin{pmatrix} -5 \\ 1 \end{pmatrix}$ (A1) (N2) 2

(ii) $\left|\overrightarrow{AB}\right| = \sqrt{25+1}$ (M1)

$= \sqrt{26}$ (= 5.10 to 3 sf) (A1) (N2) 2

***Note:** An answer of 5.1 is subject to **AP**.*

(b) $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$= \begin{pmatrix} d \\ 23 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} d-2 \\ 25 \end{pmatrix}$ (A1)(A1) 2

(c) (i) **EITHER**

$B\hat{A}D = 90° \Rightarrow \overrightarrow{AB} \bullet \overrightarrow{AD} = 0$ or mention of scalar (dot) product. (M1)

$\Rightarrow \begin{pmatrix} -5 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} d-2 \\ 25 \end{pmatrix} = 0$

$-5d + 10 + 25 = 0$ (A1)

$d = 7$ (AG)

OR

$\left.\begin{array}{l} \text{Gradient of AB} = -\frac{1}{5} \\ \text{Gradient of AD} = \frac{25}{d-2} \end{array}\right\}$ (A1)

$\left(\frac{25}{d-2}\right) \times \left(-\frac{1}{5}\right) = -1$ (A1)

$\Rightarrow d = 7$ (AG)

(ii) $\overrightarrow{OD} = \begin{pmatrix} 7 \\ 23 \end{pmatrix}$ (correct answer only) (A1) 3

(d) $\overrightarrow{AD} = \overrightarrow{BC}$ (M1)

$\overrightarrow{BC} = \begin{pmatrix} 5 \\ 25 \end{pmatrix}$ (A1)

$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$ (M1)

$\overrightarrow{OC} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} + \begin{pmatrix} 5 \\ 25 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 24 \end{pmatrix}$ (A1) (N3) 4

***Note:** Many other methods, including scale drawing, are*

acceptable.

(e) $\left|\overrightarrow{\text{AD}}\right|\left(\text{or}\left|\overrightarrow{\text{BC}}\right|\right)=\sqrt{5^2+25^2}=\sqrt{650}$ (A1)

Area = $\sqrt{26}\times\sqrt{650}$ =(5.099 × 25.5)

= 130 (A1) 2

[15]

45.) (a) (i) $\overrightarrow{\text{BC}}=\overrightarrow{\text{OC}}-\overrightarrow{\text{OB}}$

$=-6\boldsymbol{i}-2\boldsymbol{j}$ (A1)(A1) (N2)

(ii) $\overrightarrow{\text{OD}}=\overrightarrow{\text{OA}}+\overrightarrow{\text{BC}}$

$=-2\boldsymbol{i}+0\boldsymbol{j}$ ($=-2\boldsymbol{i}$) (A1)(A1) (N2)4

(b) $\overrightarrow{\text{BD}}=\overrightarrow{\text{OD}}-\overrightarrow{\text{OB}}$

$=-3\boldsymbol{i}+3\boldsymbol{j}$ (A1)

$\overrightarrow{\text{AC}}=\overrightarrow{\text{OC}}-\overrightarrow{\text{OA}}$

$=-9\boldsymbol{i}-7\boldsymbol{j}$ (A1)

Let θ be the angle between $\overrightarrow{\text{BD}}$ and $\overrightarrow{\text{AC}}$

$\cos\theta=\left(\dfrac{(-3\boldsymbol{i}+3\boldsymbol{j})\bullet(-9\boldsymbol{i}-7\boldsymbol{j})}{|(-3\boldsymbol{i}+3\boldsymbol{j})||-9\boldsymbol{i}-7\boldsymbol{j}|}\right)$ (M1)

numerator = + 27 – 21 (= 6) (A1)

denominator $=\sqrt{18}\sqrt{130}\ \left(=\sqrt{2340}\right)$ (A1)

therefore, $\cos\theta=\dfrac{6}{\sqrt{2340}}$

$\theta=82.9^{\circ}$ (1.45 rad) (A1) (N3)6

(c) $\boldsymbol{r}=\boldsymbol{i}-3\boldsymbol{j}+t(2\boldsymbol{i}+7\boldsymbol{j})\quad\left(=(1+2t)\boldsymbol{i}+(-3+7t)\boldsymbol{j}\right)$ (A1) (N1)1

(d) **EITHER**

$4\boldsymbol{i}+2\boldsymbol{j}+s(\boldsymbol{i}+4\boldsymbol{j})=\boldsymbol{i}-3\boldsymbol{j}+t(2\boldsymbol{i}+7\boldsymbol{j})$ (may be implied) (M1)

$\left.\begin{array}{l}4+s=1+2t\\2+4s=-3+7t\end{array}\right\}$ (A1)

$t=7$ and/or $s=11$ (A1)

Position vector of P is $15\boldsymbol{i}+46\boldsymbol{j}$ (A1) (N2)

OR

$7x-2y=13$ or equivalent (A1)

$4x-y=14$ or equivalent (A1)

$x=15$, $y=46$ (A1)

Position vector of P is $15\boldsymbol{i}+46\boldsymbol{j}$ (A1) (N2)4

[15]

46.) **METHOD 1**

At point of intersection:

$5 + 3\lambda = -2 + 4t$ (M1)

$1 - 2\lambda = 2 + t$ (M1)

Attempting to solve the linear system (M1)

$\lambda = -1$ (or $t = 1$) (A1)

$\overrightarrow{OP}=\begin{pmatrix}2\\3\end{pmatrix}$ (A1)(A1) (C6)

METHOD 2

(changing to Cartesian coordinates)

$2x + 3y = 13$, $x - 4y = -10$ (M1)(A1)(A1)

Attempt to solve the system (M1)

$\overrightarrow{OP}=\begin{pmatrix}2\\3\end{pmatrix}$ (A1)(A1) (C6)

Note: *Award (C5) for the point P(2, 3).*

47.) (a) $\boldsymbol{c} \cdot \boldsymbol{d} = 3 \times 5 + 4 \times (-12)$(M1)

$= -33$ (A1) (C2)

[2]

48.) (a) $\overrightarrow{OR}=\overrightarrow{PQ}$

$= \boldsymbol{q} - \boldsymbol{p}$

$=\begin{pmatrix}10\\1\end{pmatrix}-\begin{pmatrix}7\\3\end{pmatrix}$ (A1)(A1)

$=\begin{pmatrix}3\\-2\end{pmatrix}$ (A1) 3

(b) $\cos\hat{OPQ}=\dfrac{\overrightarrow{PO}\cdot\overrightarrow{PQ}}{\left|\overrightarrow{PO}\right|\times\left|\overrightarrow{PQ}\right|}$ (A1)

$\left|\overrightarrow{PO}\right|=\sqrt{(-7)^2+(-3)^2}=\sqrt{58}$, $\left|\overrightarrow{PQ}\right|=\sqrt{3^2+(-2)^2}=\sqrt{13}$ (A1)(A1)

$\overrightarrow{PO}\cdot\overrightarrow{PQ} = -21 + 6 = -15$ (A1)

$\cos\hat{OPQ}=\dfrac{-15}{\sqrt{58}\sqrt{13}}=\dfrac{-15}{\sqrt{754}}$ (AG) 4

(c) (i) Since $\hat{OPQ} + \hat{PQR} = 180°$ (R1)

$\cos\hat{PQR} = -\cos\hat{OPQ}\left(=\dfrac{15}{\sqrt{754}}\right)$ (AG)

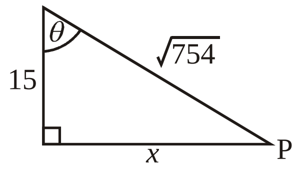
(ii) $\sin\ P\hat{Q}R = \sqrt{1-\left(\frac{15}{\sqrt{754}}\right)^2}$ (M1)

$= \sqrt{\frac{529}{754}}$ (A1)

$= \frac{23}{\sqrt{754}}$ (AG)

OR

$\cos q = \frac{15}{\sqrt{754}}$



(M1)

therefore $x^2 = 754 - 225 = 529 \Rightarrow x = 23$ (A1)

$\Rightarrow \sin q = \frac{23}{\sqrt{754}}$ (AG)

Note: *Award (A1)(A0) for the following solution.*

$\cos q = \frac{15}{\sqrt{754}} \Rightarrow q = 56.89°$

$\Rightarrow \sin q = 0.8376$

$\frac{23}{\sqrt{754}} = 0.8376 \Rightarrow \sin q = \frac{23}{\sqrt{754}}$

(iii) Area of OPQR = 2 (area of triangle PQR) (M1)

$= 2 \times \frac{1}{2}\left|\overrightarrow{PQ}\right| \times \left|\overrightarrow{QR}\right| \times \sin P\hat{Q}R$ (A1)

$= 2 \times \frac{1}{2}\sqrt{13}\sqrt{58}\frac{23}{\sqrt{754}}$ (A1)

= 23 sq units. (A1)

OR

Area of OPQR = 2 (area of triangle OPQ) (M1)

$= 2\left|\left(\frac{1}{2}\right)(7\times 1 - 3\times 10)\right|$ (A1)(A1)

= 23 sq units. (A1) 7

Notes: *Other valid methods can be used.*
Award final (A1) for the ***integer*** *answer.*

[14]

49.) B, or $\boldsymbol{r} = \begin{pmatrix}4\\4\end{pmatrix} + t\begin{pmatrix}6\\2\end{pmatrix}$ (C3)

D, or $\boldsymbol{r} = \begin{pmatrix}7\\5\end{pmatrix} + t\begin{pmatrix}3\\1\end{pmatrix}$ (C3)

Note: *Award C4 for B, D and one incorrect, C3 for one correct and nothing else, C1 for one correct and one incorrect, C0 for anything else.*

[6]

50.) (a) $\begin{pmatrix}60\\25\end{pmatrix}\bullet\begin{pmatrix}-30\\40\end{pmatrix} = 60 \times (-30) + 25 \times 40$ (M1)

$= -800$ (A1) (C2)

(b) $\cos\theta = \dfrac{-800}{\sqrt{60^2+25^2}\sqrt{(-30)^2+40^2}}$ (M1)(A1)

Note: *Trig solutions: Award M1 for attempt to use a correct strategy, A1 for correct values.*

$\cos\theta = -0.246...$ (A1)

$\theta = 104.25...°$ (or $255.75...°$) (A1) (C4)

She turns through 104° (or 256°)

Note: *Accept answers in radians ie 1.82 or 4.46.*

[6]

51.) (a) $\overrightarrow{OB} = \begin{pmatrix}-1\\7\end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix}8\\9\end{pmatrix}$ (A1)(A1) 2

(b) $\overrightarrow{AD} = \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$ (M1)

$= \begin{pmatrix}8\\9\end{pmatrix} - \begin{pmatrix}-1\\7\end{pmatrix} = \begin{pmatrix}9\\2\end{pmatrix}$ (A1)

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \begin{pmatrix}2\\2\end{pmatrix} + \begin{pmatrix}9\\2\end{pmatrix} = \begin{pmatrix}11\\4\end{pmatrix}\left(\text{or } \begin{pmatrix}8\\9\end{pmatrix} + \begin{pmatrix}3\\-5\end{pmatrix} = \begin{pmatrix}11\\4\end{pmatrix}\right)$

$d = 11 \left(\text{accept} \begin{pmatrix}11\\4\end{pmatrix}\right)$ (A1) 3

(c) $\overrightarrow{BD} = \begin{pmatrix}11\\4\end{pmatrix} - \begin{pmatrix}-1\\7\end{pmatrix} = \begin{pmatrix}12\\-3\end{pmatrix}$ (A1) 1

(d) (i) $l: \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-1\\7\end{pmatrix} + t\begin{pmatrix}12\\-3\end{pmatrix}\left(\text{or} \begin{pmatrix}-1\\7\end{pmatrix} + t\begin{pmatrix}4\\-1\end{pmatrix}\right)$ (A2)

(ii) At B, $t = 0$ by observation (A1)

OR

$\begin{pmatrix}-1\\7\end{pmatrix} = \begin{pmatrix}-1\\7\end{pmatrix} + t\begin{pmatrix}12\\-3\end{pmatrix}$

$\Rightarrow t = 0$ (A1) 3

(e) $\begin{pmatrix}7\\5\end{pmatrix} = \begin{pmatrix}-1\\7\end{pmatrix} + t\begin{pmatrix}12\\-3\end{pmatrix} \Rightarrow 7 + 1 = 12t = 8$

$\Rightarrow t = \dfrac{2}{3}$ (A1)

Note: *The equation* $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ *leads to t = 2.*

when $t = \frac{2}{3}$, $y = 7 + \left(\frac{2}{3}\right)(-3)$ (M1)

$= 7 - 2 = 5$ (A1)

ie P on line (AG)

OR

$5 - 7 = -3t = -2$

$\Rightarrow t = \frac{2}{3}$ (A1)

when $t = \frac{2}{3}$, $x = -1 + \frac{2}{3} \times 12$ (M1)

$= -1 + 8 = 7$ (A1)

ie P on line (AG) 3

(f) $\overrightarrow{CP} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} - \begin{pmatrix} 8 \\ 9 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$ (A1)

$\begin{pmatrix} -1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ -3 \end{pmatrix} = -12 + 12 = 0$ (M1)(A1)

Scalar product of non-zero vectors = 0 $\Rightarrow$ are perpendicular (R1)(AG)

OR

Geometric approach

CP: $m = 4$ (A1)

BD: $m_1 = \frac{-1}{4}$ (A1)

$mm_1 = 4 \times \left(\frac{-1}{4}\right) = -1$ (A1)

Product of gradients is –1 $\Rightarrow$ lines (vectors) are perpendicular (R1)(AG) 4

[16]

52.) $x = 1 - 2t$ (A1)

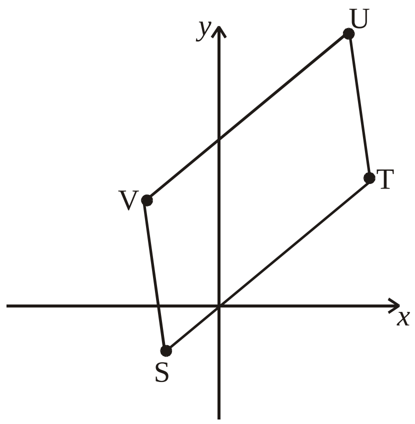
$y = 2 + 3t$ (A1)

$\frac{x-1}{-2} = \frac{y-2}{3}$ (M1)

$3x + 2y = 7$ (A1)(A1)(A1) (C6)

[6]

53.)



(a) $\overrightarrow{ST} = \boldsymbol{t} - \boldsymbol{s}$ (M1)

$= \begin{pmatrix} 7 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 9 \\ 9 \end{pmatrix}$ (A1)

$\overrightarrow{VU} = \overrightarrow{ST}$ (M1)

$\boldsymbol{u} - \boldsymbol{v} = \begin{pmatrix} 9 \\ 9 \end{pmatrix}$

$\boldsymbol{v} = \boldsymbol{u} - \begin{pmatrix} 9 \\ 9 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 15 \end{pmatrix} - \begin{pmatrix} 9 \\ 9 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$ (A1)

V(–4, 6) (A1) 5

(b) Equation of (UV): direction is = $\begin{pmatrix} 9 \\ 9 \end{pmatrix}\left(\text{or } k\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)$ (A1)

$\boldsymbol{r} = \begin{pmatrix} 5 \\ 15 \end{pmatrix} + a\begin{pmatrix} 9 \\ 9 \end{pmatrix}$ or $\begin{pmatrix} 5 \\ 15 \end{pmatrix} + a\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (A1)

OR

$\boldsymbol{r} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} + a\begin{pmatrix} 9 \\ 9 \end{pmatrix}$ or $\begin{pmatrix} -4 \\ 6 \end{pmatrix} + a\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (A1) 2

(c) $\begin{pmatrix} 1 \\ 11 \end{pmatrix}$ is on the line because it gives the same value of l , for both the *x* and *y* coordinates. (R1)

For example, $1 = 5 + 9l \quad l = -\frac{4}{9}$

$11 = 15 + 9l \quad l = -\frac{4}{9}$ (A1) 2

(d) (i) $\overrightarrow{EW} = \begin{pmatrix} a \\ 17 \end{pmatrix} - \begin{pmatrix} 1 \\ 11 \end{pmatrix}$ (M1)

$= \begin{pmatrix} a-1 \\ 6 \end{pmatrix}$ (A1)

$|\overrightarrow{EW}| = 2\sqrt{13} \Rightarrow \sqrt{(a-1)^2 + 36} = 2\sqrt{13}$ (or $(a-1)^2 + 36 = 52$)

(M1)

$a^2 - 2a + 1 + 36 = 52$

$a^2 - 2a - 15 = 0$ (A1)

$a = 5$ or $a = -3$ (A1)(AG)

(ii) For $a = -3$

$\overrightarrow{EW} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$ $\overrightarrow{ET} = \boldsymbol{t} - \boldsymbol{e} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ (A1)(A1)

$\cos \hat{WET} = \dfrac{\overrightarrow{EW} \cdot \overrightarrow{ET}}{|\overrightarrow{EW}||\overrightarrow{ET}|}$ (M1)

$= \dfrac{-24 - 24}{\sqrt{52}\sqrt{52}}$ (A1)

$= -\dfrac{12}{13}$

Therefore, $\hat{WET} = 157°$ (3 sf) (A1) 10

[19]

54.) Angle between lines = angle between direction vectors. (M1)

Direction vectors are $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ **and** $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. (A1)

$\begin{pmatrix} 4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \left|\begin{pmatrix} 4 \\ 3 \end{pmatrix}\right| \left|\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right| \cos \theta$ (M1)

$4(1) + 3(-1) = \left(\sqrt{4^2 + 3^2}\right)\left(\sqrt{1^2 + (-1)^2}\right) \cos \theta$ (A1)

$\cos \theta = \dfrac{1}{5\sqrt{2}} = 0.1414$ (A1)

$\theta = 81.9°$ (3 sf), (1.43 radians) (A1) (C6)

Note: *If candidates find the angle between the vectors* $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ *and* $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$, *award marks as below:*

Angle required is between $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ (M0)(A0)

$\begin{pmatrix} 4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \left|\begin{pmatrix} 4 \\ -1 \end{pmatrix}\right| \left|\begin{pmatrix} 2 \\ 4 \end{pmatrix}\right| \cos \theta$ (M1)

$4(2) + (-1)\,4 = \left(\sqrt{4^2 + (-1)^2}\right)\left(\sqrt{2^2 + 4^2}\right) \cos \theta$ (A1)

$\dfrac{4}{\sqrt{17}\sqrt{20}} = \cos \theta = 0.2169$ (A1)

$\theta = 77.5°$ (3sf), (1.35 radians) (A1) (C4)

[6]

55.) (i) $|\boldsymbol{a}| = \sqrt{12^2 + 5^2} = 13$ (A1)

(ii) $|\boldsymbol{b}| = \sqrt{6^2 + 8^2} = 10$ (A1)

=> unit vector in direction of $\boldsymbol{b} = \frac{1}{10}(6\boldsymbol{i} + 8\boldsymbol{j})$ (A1)

$= 0.6\boldsymbol{i} + 0.8\boldsymbol{j}$

(iii) $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$ (M1)

=> $\cos\theta = \frac{12(6) + 5(8)}{13(10)}$ (A1)

$= \frac{112}{130} = \frac{56}{65}$ (A1) 6

[6]

56.) $\cos\theta = \frac{\mathbf{a.b}}{|\mathbf{a}||\mathbf{b}|}$ (M1)

$= \frac{-4 + 14}{\sqrt{20}\sqrt{50}}$ (A1)

$= \frac{10}{10\sqrt{10}}$

$= \frac{1}{\sqrt{10}}$ (= 0.3162) (A1)

$\theta = 72°$ (to the nearest degree) (A1) (C4)

Note: *Award (C2) for a radian answer between 1.2 and 1.25.*

[4]

57.) (a) At $t = 2$, $\begin{pmatrix} 2 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0.7 \\ 1 \end{pmatrix} = \begin{pmatrix} 3.4 \\ 2 \end{pmatrix}$ (M1)

Distance from (0, 0) = $\sqrt{3.4^2 + 2^2} = 3.94$ m (A1) 2

(b) $\left|\begin{pmatrix} 0.7 \\ 1 \end{pmatrix}\right| = \sqrt{0.7^2 + 1^2}$ (M1)

$= 1.22 \text{ m s}^{-1}$ (A1) 2

(c) $x = 2 + 0.7t$ and $y = t$ (M1)

$x - 0.7y = 2$ (A1) 2

(d) $y = 0.6x + 2$ and $x - 0.7y = 2$ (M1)

$x = 5.86$ and $y = 5.52$ $\left(\text{or } x = \frac{170}{29} \text{ and } y = \frac{160}{29}\right)$ (A1)(A1) 3

(e) The time of the collision may be found by solving

$\begin{pmatrix} 5.86 \\ 5.52 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0.7 \\ 1 \end{pmatrix} t$ for t (M1)

$\Rightarrow t = 5.52$ s (A1)

[*ie* collision occurred 5.52 seconds after the vehicles set out].

Distance *d* travelled by the motorcycle is given by

$d = \left|\begin{pmatrix}5.86\\5.52\end{pmatrix} - \begin{pmatrix}0\\2\end{pmatrix}\right| = \sqrt{(5.86)^2 + (3.52)^2}$ (M1)

$= \sqrt{46.73}$

$= 6.84$ m (A1)

Speed of the motorcycle $= \dfrac{d}{t} = \dfrac{6.84}{5.52}$

$= 1.24$ m s^{-1} (A1) 5

[14]

58.) Direction vector $= \begin{pmatrix}6\\5\end{pmatrix} - \begin{pmatrix}1\\3\end{pmatrix}$ (M1)

$= \begin{pmatrix}5\\2\end{pmatrix}$ (A1)

$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix} + t\begin{pmatrix}5\\2\end{pmatrix}$ (A2)

OR

$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}6\\5\end{pmatrix} + t\begin{pmatrix}5\\2\end{pmatrix}$ (A2) (C4)

[4]

59.) (a) $\begin{pmatrix}2x\\x-3\end{pmatrix} \bullet \begin{pmatrix}x+1\\5\end{pmatrix} = 0$ (M1)(M1)

$\Rightarrow 2x(x + 1) + (x - 3)(5) = 0$ (A1)

$\Rightarrow 2x^2 + 7x - 15 = 0$ (C3)

(b) **METHOD 1**

$2x^2 + 7x - 15 = (2x - 3)(x + 5) = 0$

$\Rightarrow x = \dfrac{3}{2}$ or $x = -5$ (A1) (C1)

METHOD 2

$x = \dfrac{-7 \pm \sqrt{7^2 - 4(2)(-15)}}{2(2)}$

$\Rightarrow x = \dfrac{3}{2}$ or $x = -5$ (A1) (C1)

[4]

60.) (a) (i) $\overrightarrow{OA} = \begin{pmatrix}240\\70\end{pmatrix}$ OA $= \sqrt{240^2 + 70^2} = 250$ (A1)

unit vector = $\frac{1}{250}\begin{pmatrix}240\\70\end{pmatrix} = \begin{pmatrix}0.96\\0.28\end{pmatrix}$ (M1)(AG)

(ii) $\bar{\boldsymbol{v}} = 300\begin{pmatrix}0.96\\0.28\end{pmatrix} = \begin{pmatrix}288\\84\end{pmatrix}$ (M1)(A1)

(iii) $t = \frac{240}{288} = \frac{5}{6}$ hr (= 50 min) (A1) 5

(b) $\overrightarrow{\text{AB}} = \begin{pmatrix}480-240\\250-70\end{pmatrix} = \begin{pmatrix}240\\180\end{pmatrix}$ (A1)

AB = $\sqrt{240^2+180^2}$ = 300

$\cos\theta = \frac{\overrightarrow{\text{OA}}\bullet\overrightarrow{\text{AB}}}{\text{OA}\times\text{AB}} = \frac{(240)(240)+(70)(180)}{(250)(300)}$ (M1)

= 0.936 (A1)

$\Rightarrow \theta = 20.6°$ (A1) 4

(c) (i) $\overrightarrow{\text{AX}} = \begin{pmatrix}339-240\\238-70\end{pmatrix} = \begin{pmatrix}99\\168\end{pmatrix}$ (A1)

(ii) $\begin{pmatrix}-3\\4\end{pmatrix}\bullet\begin{pmatrix}240\\180\end{pmatrix}$ = –720 + 720 = 0 (M1)(A1)

$\Rightarrow \boldsymbol{n} \perp \overrightarrow{\text{AB}}$ (AG)

(iii) Projection of $\overrightarrow{\text{AX}}$ in the direction of $\boldsymbol{n}$ is

XY = $\frac{1}{5}\begin{pmatrix}99\\168\end{pmatrix}\bullet\begin{pmatrix}-3\\4\end{pmatrix} = \frac{-297+672}{5}$ = 75 (M1)(A1)(A1) 6

(d) AX = $\sqrt{99^2+168^2}$ = 195 (A1)

AY = $\sqrt{195^2-75^2}$ = 180 km (M1)(A1) 3

[18]

61.) $\begin{pmatrix}1\\2\end{pmatrix}\cdot\begin{pmatrix}6\\-8\end{pmatrix}$ = 6 – 16 = –10 (A1)

$\left|\begin{pmatrix}1\\2\end{pmatrix}\right| = \sqrt{1^2+2^2} = \sqrt{5}, \left|\begin{pmatrix}6\\-8\end{pmatrix}\right| = \sqrt{6^2+8^2} = \sqrt{100} = 10$ (A1)

$\begin{pmatrix}1\\2\end{pmatrix}\cdot\begin{pmatrix}6\\-8\end{pmatrix} = \left|\begin{pmatrix}1\\2\end{pmatrix}\right|\left|\begin{pmatrix}6\\-8\end{pmatrix}\right|\cos q$

$-10 = \sqrt{5} \times 10\cos q \Rightarrow \cos q = \frac{-10}{10\sqrt{5}} = -\frac{1}{\sqrt{5}} \Rightarrow q = \arccos\frac{-1}{\sqrt{5}}$ (M1)

$q \approx 117°$ (A1)

[4]

62.) $\begin{pmatrix}2\\3\end{pmatrix}\cdot\begin{pmatrix}x-4\\y+1\end{pmatrix}$ (M1) (M1)

Notes: *Award (M1) for using scalar product.*

Award (M1) for $\begin{pmatrix} x-4 \\ y+1 \end{pmatrix}$.

$2(x – 4) + 3(y + 1) = 0$ (A1)

$2x – 8 + 3y + 3 = 0$

$2x + 3y = 5$ (A1)

OR

Gradient of a line parallel to the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is $\frac{3}{2}$ (M1)

Gradient of a line perpendicular to this line is $-\frac{2}{3}$ (M1)

So the equation is $y + 1 = -\frac{2}{3}(x – 4)$ (A1)

$\Rightarrow 3y + 3 = –2x + 8$

$\Rightarrow 2x + 3y = 5$ (A1)

[4]

63.) (a) At 13:00, $t = 1$ (M1)

$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 28 \end{pmatrix} + 1 \times \begin{pmatrix} 6 \\ -8 \end{pmatrix} = \begin{pmatrix} 6 \\ 20 \end{pmatrix}$ (A1) 2

(b) (i) Velocity vector: $\begin{pmatrix} x \\ y \end{pmatrix}_{t=1} - \begin{pmatrix} x \\ y \end{pmatrix}_{t=0}$ (M1)

$= \begin{pmatrix} 6 \\ 20 \end{pmatrix} - \begin{pmatrix} 0 \\ 28 \end{pmatrix} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$ (km h^{-1})(A1)

(ii) Speed = $\sqrt{(6^2 + (-8)^2)}$; (M1)

= 10; 10 km h^{-1} (A1) 4

(c) **EITHER** $\left.\begin{aligned} x &= 6t \\ y &= 28 - 8t \end{aligned}\right\}$ (M1)

Note: *Award (M1) for both equations.*

$\Rightarrow y = 28 – 8\left(\frac{x}{6}\right)$ (M1)(A1)

Note: *Award (M1) for elimination, award (A1) for equation in x, y.*

$\Rightarrow 4x + 3y = 84$ (a1) 4

OR

$\begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -8 \end{pmatrix}^{\perp} = \begin{pmatrix} 0 \\ 28 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -8 \end{pmatrix}^{\perp}$ (M1)

$\begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 28 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 6 \end{pmatrix}$ (M1)(A1)

$\Leftrightarrow 4x + 3y = 84$ (A1) 4

(d) They collide if $\begin{pmatrix}18\\4\end{pmatrix}$ lies on path; (R1)

EITHER (18, 4) lies on $4x + 3y = 84$

$\Leftrightarrow 4 \times 18 + 3 \times 4 = 84$

$\Leftrightarrow 72 + 12 = 84$; OK; (M1)

$x = 18$ (M1)

$\Rightarrow 18 = 6t \Rightarrow t = 3$, collide at 15:00 (A1) 4

OR $\begin{pmatrix}18\\4\end{pmatrix} = \begin{pmatrix}0\\28\end{pmatrix} + t\begin{pmatrix}6\\-8\end{pmatrix}$ for some t,

$\Leftrightarrow \left\{\begin{matrix}18 = 6t\\ \textbf{and } 4 = 28 - 8t\end{matrix}\right\}$ (A1)

$\Leftrightarrow \left\{\begin{matrix}t = 3\\ \textbf{and } 8t = 24\end{matrix}\right\}$ (A1)

$\Leftrightarrow \left\{\begin{matrix}t = 3\\ \textbf{and } t = 3\end{matrix}\right\}$

They collide at 15:00 (A1) 4

(e) $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}18\\4\end{pmatrix} + (t-1)\begin{pmatrix}5\\12\end{pmatrix}$ (M1)

$= \begin{pmatrix}18 + 5t - 5\\4 + 12t - 12\end{pmatrix}$ (M1) 2

$= \begin{pmatrix}13\\-8\end{pmatrix} + t\begin{pmatrix}5\\12\end{pmatrix}$ (AG)

(f) At $t = 3$, (M1)

$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}13 + 3\times 5\\-8 + 3\times 12\end{pmatrix} = \begin{pmatrix}28\\28\end{pmatrix}$ (A1)

$\begin{pmatrix}28\\28\end{pmatrix} - \begin{pmatrix}18\\4\end{pmatrix} = \begin{pmatrix}10\\24\end{pmatrix}$ (A1)

$\sqrt{(10^2 + 24^2)} = \sqrt{(676)} = 26$

26 km apart (A1) 4

[20]

64.) $\boldsymbol{u} + \boldsymbol{v} = 4\boldsymbol{i} + 3\boldsymbol{j}$ (A1)

Then $a(4\boldsymbol{i} + 3\boldsymbol{j}) = 8\boldsymbol{i} + (b-2)\boldsymbol{j}$

$4a = 8$

$3a = b - 2$ (A1)

Whence $a = 2$ (A1) (C2)

$b = 8$ (A1) (C2)

[4]

65.) Required vector will be parallel to $\begin{pmatrix}3\\-1\end{pmatrix} - \begin{pmatrix}-1\\4\end{pmatrix}$ (M1)

$= \begin{pmatrix}4\\-5\end{pmatrix}$ (A1)

Hence required equation is $\boldsymbol{r} = \begin{pmatrix}-1\\4\end{pmatrix} + t\begin{pmatrix}4\\-5\end{pmatrix}$ (A1)(A1) (C4)

Note: *Accept alternative answers, eg* $\begin{pmatrix}3\\-1\end{pmatrix} + s\begin{pmatrix}4\\-5\end{pmatrix}$.

[4]

66.) (a) $\left|\begin{pmatrix}18\\24\end{pmatrix}\right| = 30 \text{ km h}^{-1}$ (A1)

$\left|\begin{pmatrix}36\\-16\end{pmatrix}\right| = \sqrt{36^2 + (-16)^2}$

$= 39.4$ (A1) 2

(b) (i) After ½ hour, position vectors are

$\begin{pmatrix}9\\12\end{pmatrix}$ and $\begin{pmatrix}18\\-8\end{pmatrix}$ (A1)(A1)

(ii) At 6.30 am, vector joining their positions is

$\begin{pmatrix}9\\12\end{pmatrix} - \begin{pmatrix}18\\-8\end{pmatrix} = \begin{pmatrix}-9\\20\end{pmatrix}$ (or $\begin{pmatrix}9\\-20\end{pmatrix}$) (M1)

$\left|\begin{pmatrix}-9\\20\end{pmatrix}\right|$ (M1)

$= \sqrt{481}$ (= 21.9 km to 3 sf) (A1) 5

(c) The Toyundai must continue until its position vector is $\begin{pmatrix}18\\k\end{pmatrix}$ (M1)

Clearly $k = 24$, *ie* position vector $\begin{pmatrix}18\\24\end{pmatrix}$. (A1)

To reach this position, it must travel for 1 hour in total. (A1)
Hence the crew starts work at 7.00 am (A1) 4

(d) Southern (Chryssault) crew lays 800 × 5 = 4000 m (A1)
Northern (Toyundai) crew lays 800 × 4.5 = 3600 m (A1)
Total by 11.30 am = 7.6 km

Their starting points were 24 – (–8) = 32 km apart (A1)
Hence they are now 32 – 7.6 = 24.4 km apart (A1) 4

(e) Position vector of Northern crew at 11.30 am is

$\begin{pmatrix}18\\24-3.6\end{pmatrix} = \begin{pmatrix}18\\20.4\end{pmatrix}$ (M1)(A1)

Distance to base camp = $\left|\begin{pmatrix}18\\20.4\end{pmatrix}\right|$ (A1)

$= 27.2$ km

Time to cover this distance = $\frac{27.2}{30} \times 60$ (A1)

= 54.4 minutes
= 54 minutes (to the nearest minute) (A1) 5

[20]

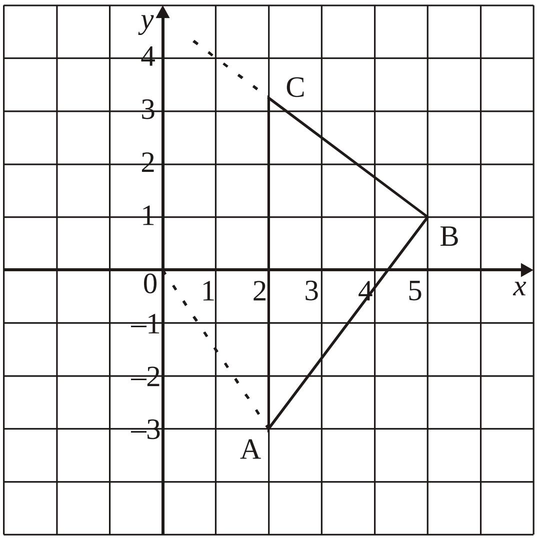
67.) Vector equation of a line $\boldsymbol{r} = \boldsymbol{a} + \mathrm{l}\, \boldsymbol{t}$ (M1)

$\boldsymbol{a} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\boldsymbol{t} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ (M1)(M1)

$\Rightarrow \boldsymbol{r} = \mathrm{l}\,(2\boldsymbol{i} + 3\boldsymbol{j})$ (A1) (C4)

[4]

68.) (a)



(A3) (C3)

Note: *Award (A1) for B at (5, 1); (A1) for BC perpendicular to AB; (A1) for AC parallel to the y-axis.*

(b) $\overrightarrow{OC} = \begin{pmatrix} 2 \\ 3.25 \end{pmatrix}$ (A1) (C1)

Note: *Accept correct readings from diagram (allow ±0.1).*

[4]

69.) (a) (i) $\boldsymbol{r}_1 = \begin{bmatrix} 16 \\ 12 \end{bmatrix} + t\begin{bmatrix} 12 \\ -5 \end{bmatrix}$

$t = 0 \Rightarrow \boldsymbol{r}_1 = \begin{bmatrix} 16 \\ 12 \end{bmatrix}$ (M1)

$|\boldsymbol{r}_1| = \sqrt{(16^2 + 12^2)} = 20$ (A1)

(ii) Velocity vector = $\begin{bmatrix} 12 \\ -5 \end{bmatrix}$

$\Rightarrow$ speed = $\sqrt{(12^2 + (-5)^2)}$ (M1)

$= 13$ (A1) 4

(b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \end{bmatrix} + t\begin{bmatrix} 12 \\ -5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix}5\\12\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}5\\12\end{bmatrix}\cdot\begin{bmatrix}6\\12\end{bmatrix} + \begin{bmatrix}5\\12\end{bmatrix}\cdot t\begin{bmatrix}12\\-5\end{bmatrix}$ (M1)

$\Rightarrow 5x + 12y = 80 + 144$ (A1)

$5x + 12y = 224$ (A1)(AG)

OR

$\dfrac{x-16}{12} = \dfrac{y-12}{-5}$ (M1)

$5x - 80 = 144 - 12y$ (A1)

$\Rightarrow 5x + 12y = 224$ (A1)(AG)

OR

$x = 16 + 12t,\ y = 12 - 5t \Rightarrow t = \dfrac{12-y}{5}$ (M1)

$\Rightarrow x = 16 + 12\left(\dfrac{12-y}{5}\right)$ (A1)

$\Rightarrow 5x = 80 + 144 - 12y$

$\Rightarrow 5x + 12y = 224$ (A1)(AG) 3

(c) $\boldsymbol{v}_1 = \begin{bmatrix}12\\-5\end{bmatrix} \qquad \boldsymbol{v}_2 = \begin{bmatrix}2.5\\6\end{bmatrix}$ (M1)

$\boldsymbol{v}_1.\boldsymbol{v}_2 = \begin{bmatrix}12\\-5\end{bmatrix}\cdot\begin{bmatrix}2.5\\6\end{bmatrix}$ (M1)

$= 30 - 30$

$\Rightarrow \boldsymbol{v}_1.\boldsymbol{v}_2 = 0$ (A1)

$\Rightarrow \theta = 90°$ (A1) 4

(d) (i) $\begin{bmatrix}x\\y\end{bmatrix}\cdot\begin{bmatrix}12\\-5\end{bmatrix} = \begin{bmatrix}23\\-5\end{bmatrix}\cdot\begin{bmatrix}12\\-5\end{bmatrix}$ (M1)

$\Rightarrow 12x - 5y = 23 \times 12 + 25 = 301$ (A1)

OR

$\dfrac{x-23}{2.5} = \dfrac{y+5}{6}$

$\Rightarrow 6x - 138 = 2.5y + 12.5$ (M1)

$\Rightarrow 12x - 276 = 5y + 25$

$\Rightarrow 12x - 5y = 301$ (A1)

(ii) $\left.\begin{array}{l}5x + 12y = 224\\12x - 5y = 301\end{array}\right\} \Rightarrow \left.\begin{array}{l}25x + 60y = 1120\\144x - 60y = 3612\end{array}\right\}$ (M1)

$169x = 4732$

$x = 28,\ y = (12 \times 28 - 301) \div 5 = 7$

$(28, 7)$ (A1)(A1) 5

Note: *Accept any correct method for solving simultaneous equations.*

(e) $16 + 12t = 23 + 2.5t \Rightarrow 9.5t = 7$ (M1)

$12 - 5t = -5 + 6t \quad \Rightarrow 17 = 11t$ (M1)

$\dfrac{7}{9.5} \neq \dfrac{17}{11}$ (A1)

$\Rightarrow$ planes cannot be at the same place at the same time (R1)

OR

$\boldsymbol{r}_1 = \begin{bmatrix} 28 \\ 7 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 28 \\ 7 \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \end{bmatrix} + t\begin{bmatrix} 12 \\ -5 \end{bmatrix}$ (M1)

$\Leftrightarrow \begin{cases} 12t = 12 \\ -5t = -5 \end{cases} \Leftrightarrow t = 1$ (A1)

When $t = 1$ $\boldsymbol{r}_2 = \begin{bmatrix} 23 \\ -5 \end{bmatrix} + \begin{bmatrix} 2.5 \\ 6 \end{bmatrix} = \begin{bmatrix} 25.5 \\ 1 \end{bmatrix} \neq \begin{bmatrix} 28 \\ 7 \end{bmatrix}$ (A1)(R1)

OR

$\boldsymbol{r}_2 = \begin{bmatrix} 28 \\ 7 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 28 \\ 7 \end{bmatrix} = \begin{bmatrix} 23 \\ -5 \end{bmatrix} + t\begin{bmatrix} 2.5 \\ 6 \end{bmatrix}$ (M1)

$\Leftrightarrow t = 2$ (A1) 4

[20]

70.) (a) $\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$ (A1) (C1)

(b) $\overrightarrow{OA} = \frac{1}{2}\overrightarrow{CD}$

$= \frac{1}{2}(\overrightarrow{OD} - \overrightarrow{OC})$ (A1) (C1)

(c) $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$= \overrightarrow{OD} - \frac{1}{2}(\overrightarrow{OD} - \overrightarrow{OC})$ (A1)

$= \frac{1}{2}\overrightarrow{OD} + \frac{1}{2}\overrightarrow{OC}$ (A1) (C2)

Note: *Deduct [1 mark] (once only) if appropriate vector notation is omitted.*

[4]

71.) (a) $u = -i + 2j$ $v = 3i + 5j$

$u + 2v = 5i + 12j$ (A1) (C1)

(b) $|\vec{u} + 2\vec{v}| = \sqrt{5^2 + 12^2}$

$= 13$ (A1)

Vector $\vec{w} = \frac{26}{13}(5\vec{i} + 12\vec{j})$ (A1)

$= 10i + 24j$ (A1) (C3)

[4]

72.) (a) $|\overrightarrow{OA}| = 6$ $\Rightarrow$ A is on the circle (A1)

$|\overrightarrow{OB}| = 6$ $\Rightarrow$ B is on the circle. (A1)

$\left|\overrightarrow{OC}\right| = \left|\begin{pmatrix} 5 \\ \sqrt{11} \end{pmatrix}\right|$

$= \sqrt{25+11}$

$= 6 \quad \Rightarrow \quad C$ is on the circle. (A1) 3

(b) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= \begin{pmatrix} 5 \\ \sqrt{11} \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ (M1)

$= \begin{pmatrix} -1 \\ \sqrt{11} \end{pmatrix}$ (A1) 2

(c) $\cos O\hat{A}C = \dfrac{\overrightarrow{AO} \cdot \overrightarrow{AC}}{\left|AO\right|\left|AC\right|}$ (M1)

$= \dfrac{\begin{pmatrix} -6 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ \sqrt{11} \end{pmatrix}}{6\sqrt{1+11}}$

$= \dfrac{6}{6\sqrt{12}}$ (A1)

$= \dfrac{1}{2\sqrt{3}} = \dfrac{\sqrt{3}}{6}$ (A1)

OR $\cos O\hat{A}C = \dfrac{6^2 + (\sqrt{12})^2 - 6^2}{2 \times 6 \times \sqrt{12}}$ (M1)(A1)

$= \dfrac{1}{\sqrt{12}}$ as before (A1)

OR using the triangle formed by $\overrightarrow{AC}$ and its horizontal and vertical components:

$\left|\overrightarrow{AC}\right| = \sqrt{12}$ (A1)

$\cos O\hat{A}C = \dfrac{1}{\sqrt{12}}$ (M1)(A1) 3

Note: *The answer is 0.289 to 3 sf*

(d) A number of possible methods here

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= \begin{pmatrix} 5 \\ \sqrt{11} \end{pmatrix} - \begin{pmatrix} -6 \\ 0 \end{pmatrix}$ (A1)

$= \begin{pmatrix} 11 \\ \sqrt{11} \end{pmatrix}$ (A1)

$|BC| = \sqrt{132}$

$|\Delta ABC| = \dfrac{1}{2} \times \sqrt{132} \times \sqrt{12}$ (A1)

$= 6\sqrt{11}$ (A1)

OR ΔABC has base $AB = 12$ (A1)

and height $= \sqrt{11}$ (A1)

$\Rightarrow$ area = $\frac{1}{2} \times 12 \times \sqrt{11}$ (A1)

= $6\sqrt{11}$ (A1)

OR Given $\cos B\hat{A}C = \frac{\sqrt{3}}{6}$

$\sin B\hat{A}C = \frac{\sqrt{33}}{6} \Rightarrow |\Delta ABC| = \frac{1}{2} \times 12 \times \sqrt{12} \times \frac{\sqrt{33}}{6}$ (A1)(A1)(A1)

= $6\sqrt{11}$ (A1) 4

[12]

73.) (a) $\overrightarrow{OB} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ (A1) (C1)

$\overrightarrow{AC} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$ (A1) (C1)

(b) $\overrightarrow{OB} \cdot \overrightarrow{AC} = (10 \times (-3)) + (5 \times 6) = 0$ (M1)

Angle = 90° (A1) (C2)

[4]